Spatial encoding in Magnetic Resonance Imaging

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What's an image ?

« … a reproduction of a material object by a camera or a related technique »

Multi-dimensional signal

I(**r**) **r** = vector of spatial coordinates $(x,y,z)^T$

Signal intensity of the image

I is scalar = Image with a single frame

I is a vector = Multi-channel image

The first MR image

Made by Paul LAUTERBUR, Nature (1973)

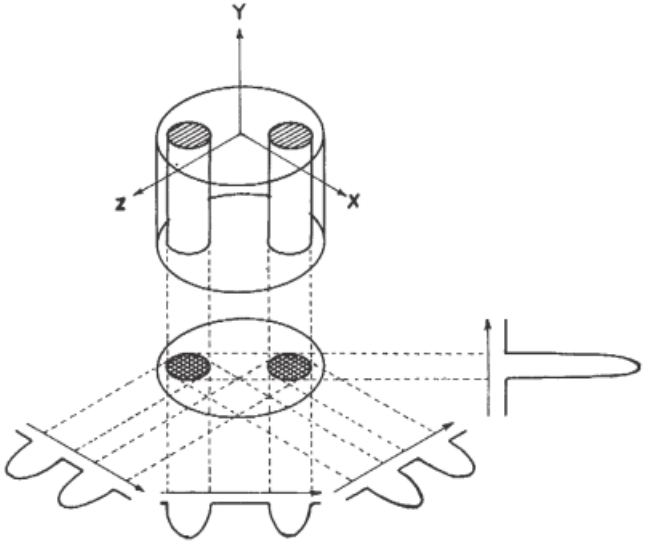
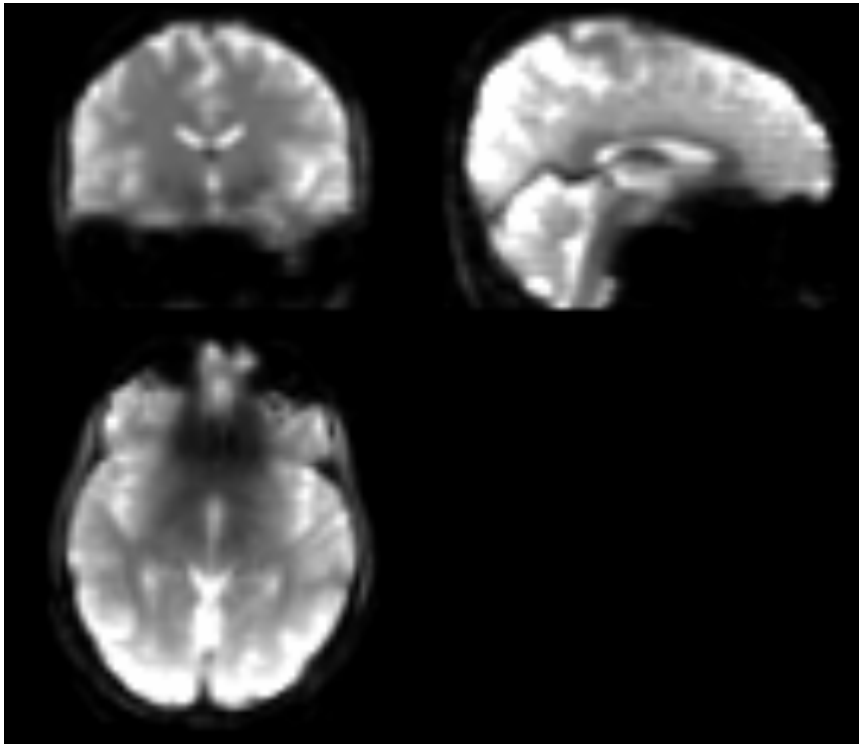


Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Fig. 2 Proton nuclear magnetic resonance zeugmatogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.

There are 3 spatial dimensions

In *real* world, images should be in 3D

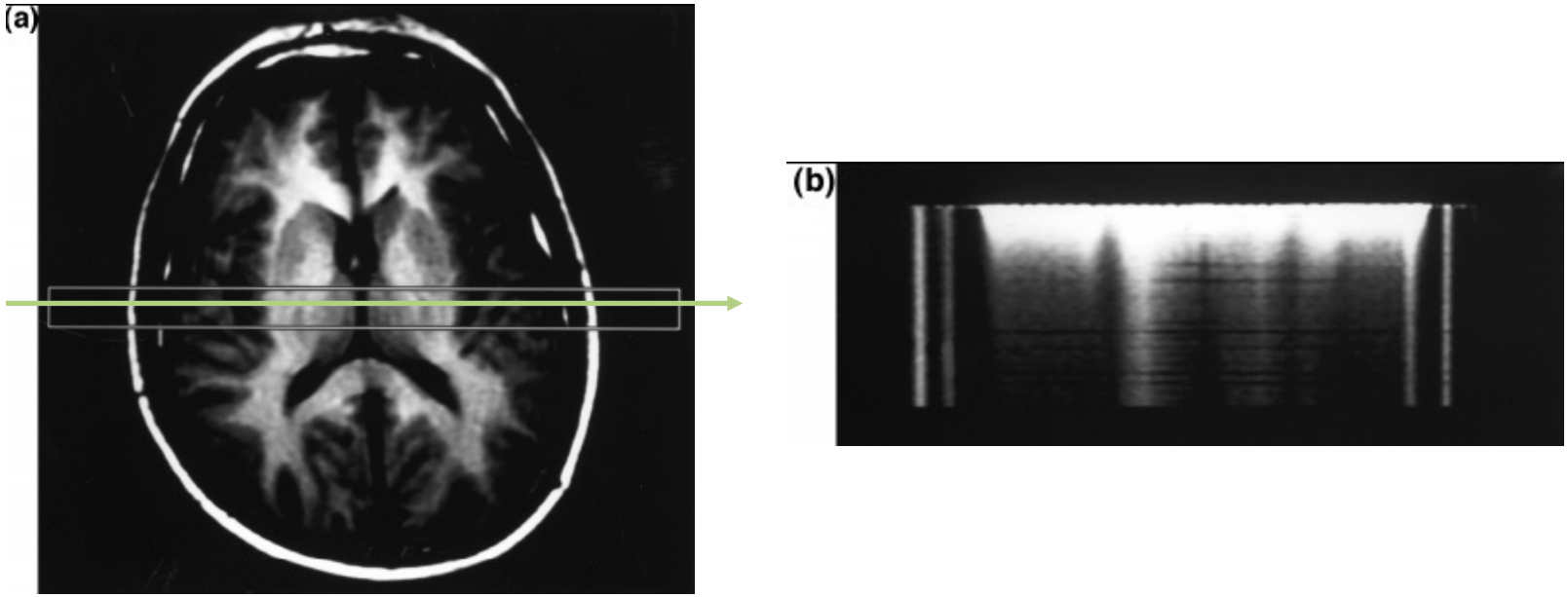


Human brain – V = 3 x 3 x 3 mm^3

1D or 2D images

The concept of « profile »

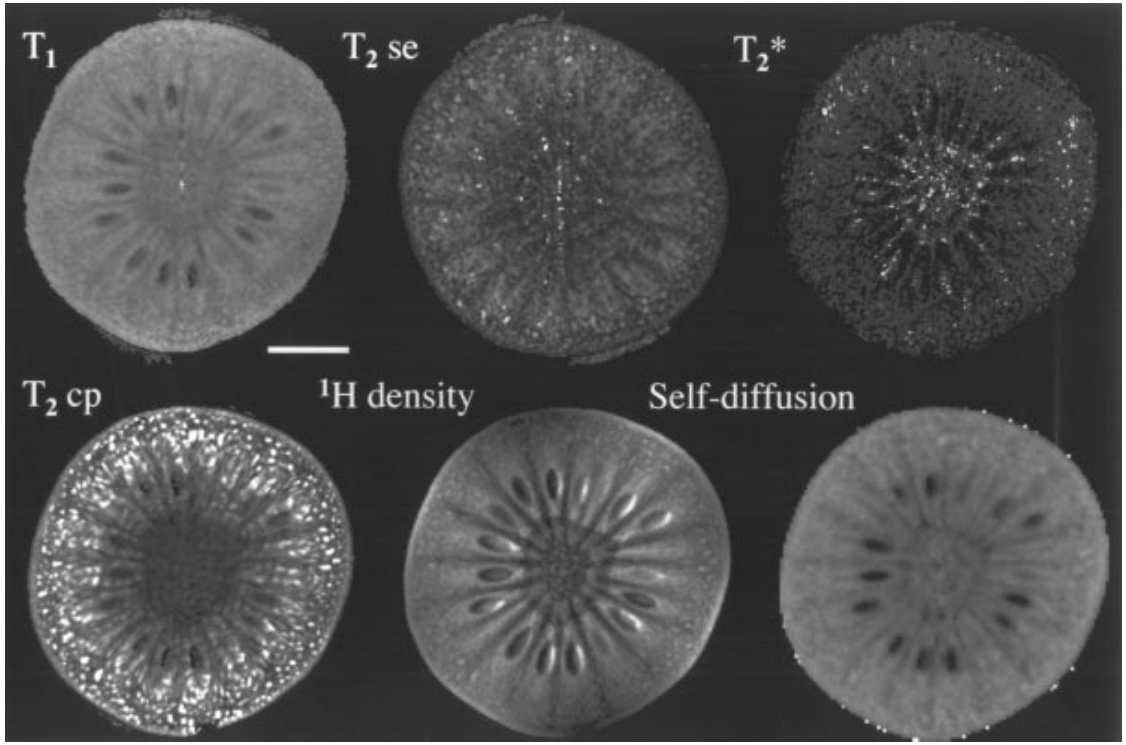
$$\mathbf{I}(x) = \int \mathbf{I}(x, y, z)\, dy\, dz$$



Mulkern *et al*, NMR Biomed (1999) 12:51-62

Multi-channel images ?

MRI provides different contrasts



Clark *et al*, J Sci Food Agric (1998) 78

The MRI challenges

More sensitivity

Improvement of spatial and temporal resolutions

More specificity

Tissue discrimination through the signature given by the signal I

How to encode spatial information ?

The target

Amplitude M_T of transverse magnetization over space

$$M_T(\mathbf{r})$$

r : Spatial Cartesian coordinates

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

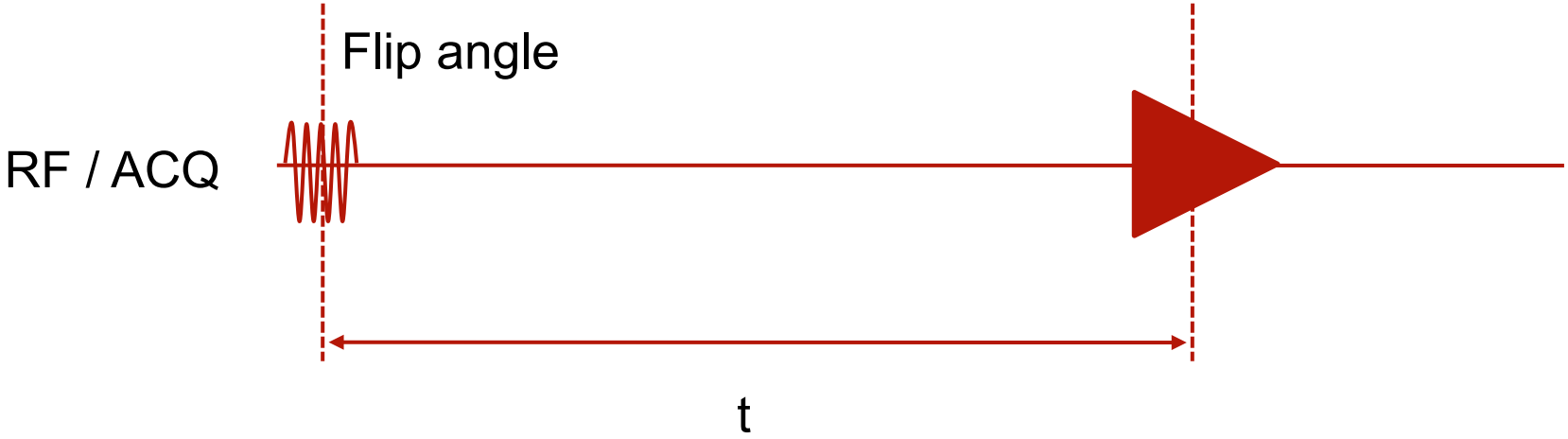
Assumptions

No relaxation

Uncoupled spins

Signal without encoding

Free induction decay



Signal without encoding

FID

$$\hat{S}(\mathbf{r},t) = M_T(\mathbf{r})\exp\left[i\phi(\mathbf{r},t)\right]$$

$\hat{S}$ Complex signal **because of quadrature detection**

$$\phi(\mathbf{r},t) = 2\pi\int_0^t \left[f(\mathbf{r},t') - f_0\right]dt'$$

ϕ Phase accumulated during *t* period

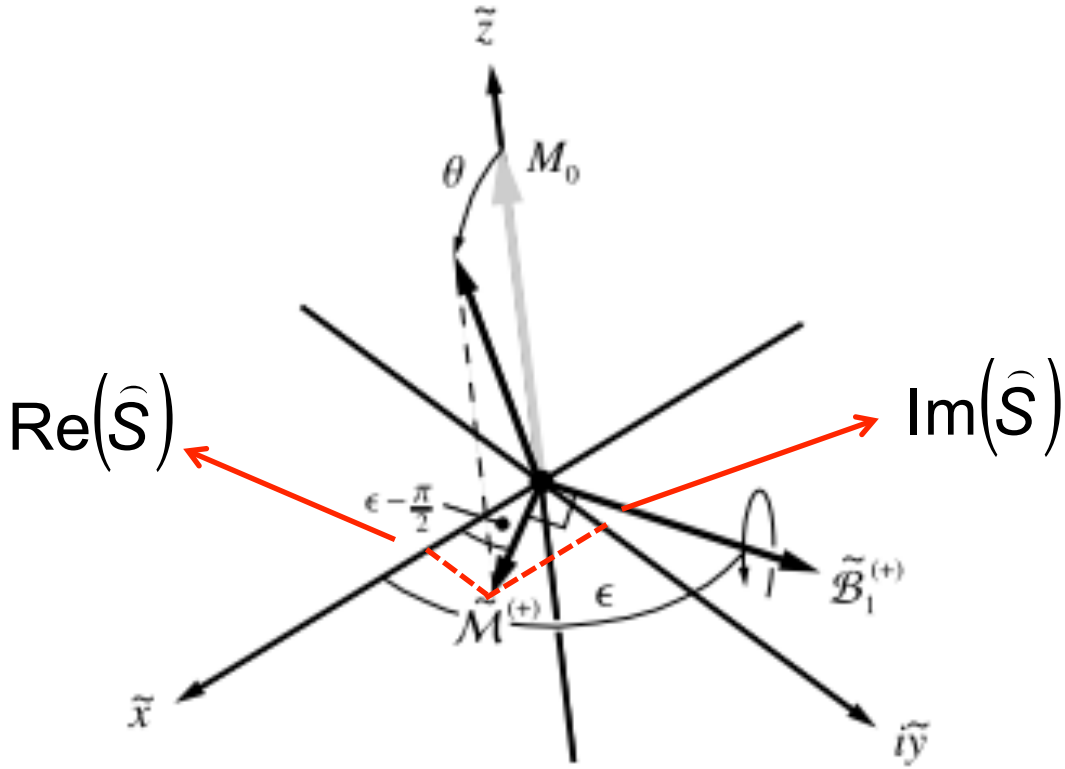
On-resonance signal coming from the whole sample

$$\hat{S} = \int \hat{S}(\mathbf{r},t)d\mathbf{r} = \int M_T(\mathbf{r})d\mathbf{r}$$

Quadrature detection

Components of M_T in the rotating frame

Introduce phase/off-resonance information



Hoult, *Concepts in Magnetic Resonance* (2000)

What is a gradient ?

Mathematical operator

Vector of partial derivatives with respect to each coordinate

For a 3D function $fun(\mathbf{r}) = fun(x, y, z)$

$$\text{grad}\, fun(\mathbf{r}) = \nabla fun(\mathbf{r}) = \begin{bmatrix} \dfrac{\partial fun(\mathbf{r})}{\partial x} \\ \dfrac{\partial fun(\mathbf{r})}{\partial y} \\ \dfrac{\partial fun(\mathbf{r})}{\partial z} \end{bmatrix}$$

fun = magnetic field (MF) *B*

Linearly varying magnetic fields $G_{x,y,z}$ in each direction

G is **superposed** to the static field (B_0)

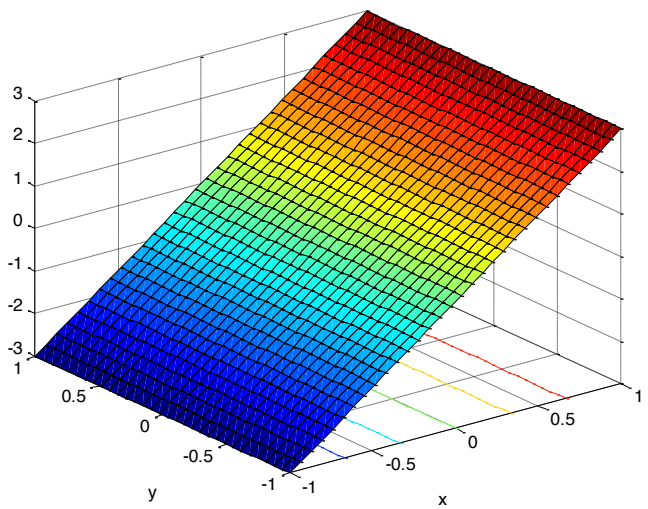
$$B(\mathbf{r}) = B_0 \quad \Rightarrow \quad \begin{aligned} B(\mathbf{r}) &= B_0 + G_x.x + G_y.y + G_z.z \\ &= B_0 + \mathbf{G}.\mathbf{r} \end{aligned}$$

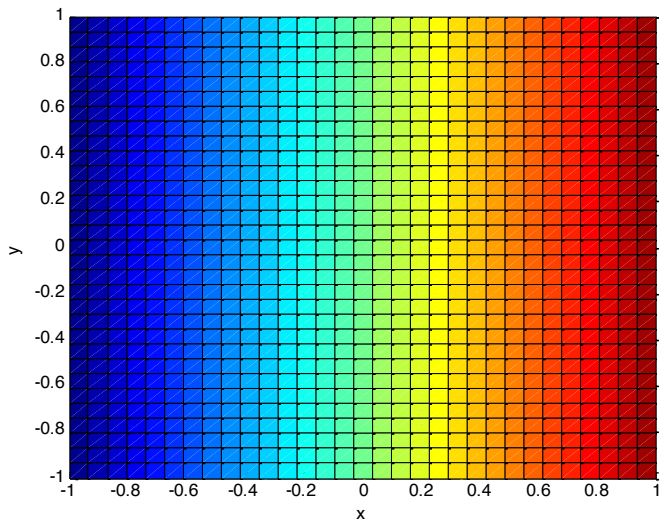
$$\mathbf{G} = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix}$$

Some examples of linearly varying MF

$$B(\mathbf{r}) = B_0 + G_x.x + G_y.y + G_z.z$$

$$G_x = 3 \quad G_y = 0 \quad z = 0$$

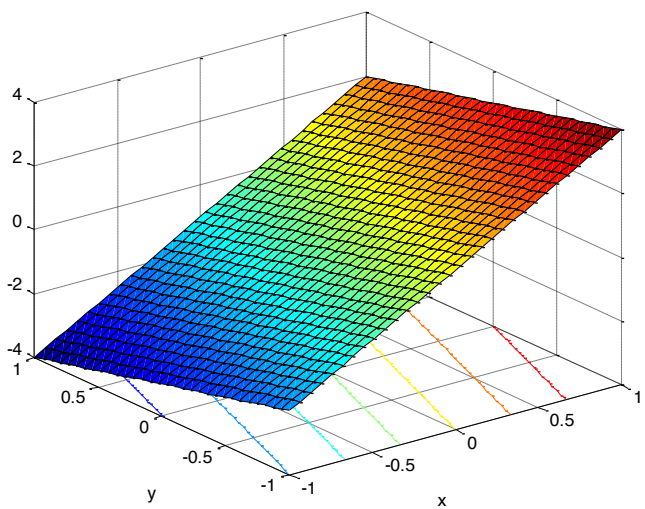


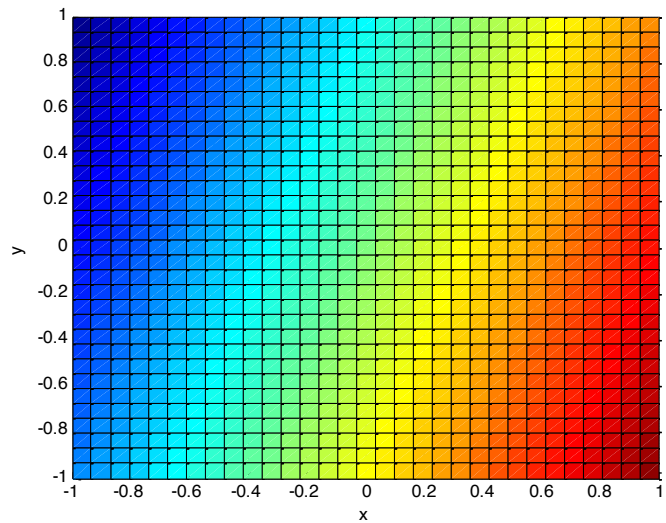


Some examples of linearly varying MF

$$B(\mathbf{r}) = B_0 + G_x.x + G_y.y + G_z.z$$

$$G_x = 3 \quad G_y = -1 \quad z = 0$$



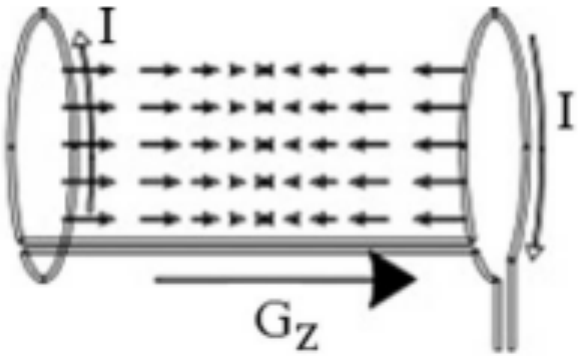


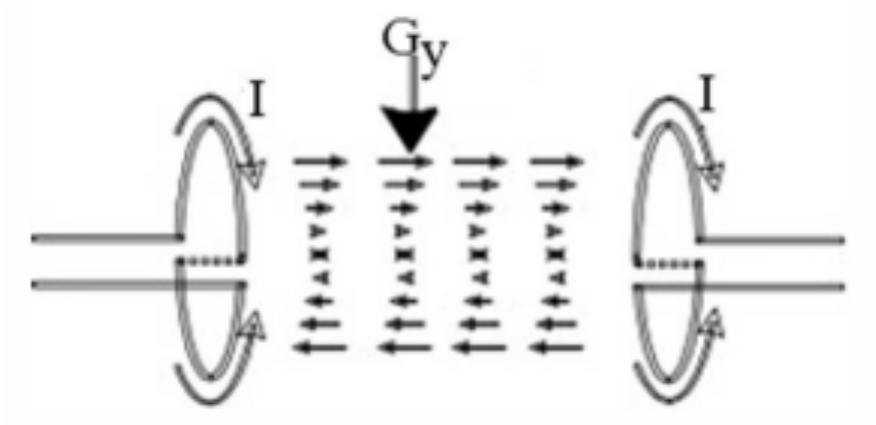
Gradient of the magnetic field

$$\nabla B(\mathbf{r}) = \begin{bmatrix} \dfrac{\partial\left(B_0 + G_x.x + G_y.y + G_z.z\right)}{\partial x} \\ \dfrac{\partial\left(B_0 + G_x.x + G_y.y + G_z.z\right)}{\partial y} \\ \dfrac{\partial\left(B_0 + G_x.x + G_y.y + G_z.z\right)}{\partial z} \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \mathbf{G}$$

G corresponds to the gradient of the MF
The *gradient coils* generate **G**

Gradient coils / Principles





Hidalgo-Tobon, *Concepts in Magnetic Resonance* (2010)

Gradient coils / How it looks like ?

Effect of currents

The Biot-Savart law (1820)

MF generated by a current that circulates through wires

$$d\mathbf{B} = \frac{\mu_0}{4\pi} I \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

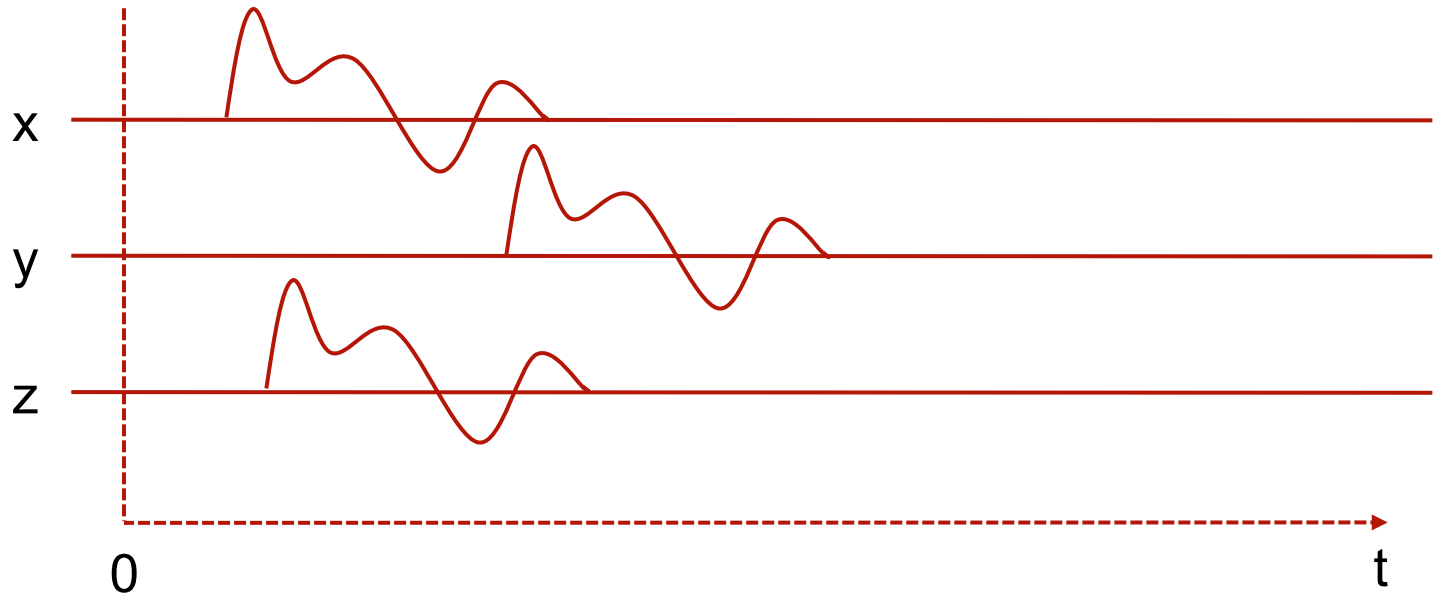
I = current intensity

$$B \propto I$$

Time varying fields

Currents = independant functions of time

$$\mathbf{I}(t) = \begin{bmatrix} I_x(t) \\ I_y(t) \\ I_z(t) \end{bmatrix} \Rightarrow \mathbf{G}(t) = \begin{bmatrix} G_x(t) \\ G_y(t) \\ G_z(t) \end{bmatrix}$$



Effects of MF gradients on NMR signal

Time AND spatially dependent frequency

$$B(\mathbf{r}) = B_0 + \mathbf{G}.\mathbf{r} \quad \Rightarrow \quad B(\mathbf{r},t) = B_0 + \mathbf{G}(t).\mathbf{r}$$

Larmor equation

$$f(\mathbf{r},t) = \frac{\gamma}{2\pi} B_0 + \frac{\gamma}{2\pi} \mathbf{G}(t).\mathbf{r}$$

Phase of the transverse magnetization

$$\phi(\mathbf{r},t) = 2\pi \int_0^t \left[f(\mathbf{r},t') - f_0 \right] dt' = \gamma \int_0^t \mathbf{G}(t').\mathbf{r}\, dt'$$

The k variable

Gradient antiderivatives

$$\phi(\mathbf{r},t) = \gamma\int_0^t \mathbf{G}(t').\mathbf{r}dt' = \mathbf{r}.\gamma\int_0^t \mathbf{G}(t')dt' = \mathbf{r}.\mathbf{k}(t)$$

$$\mathbf{k}(t) = \gamma\int_0^t \mathbf{G}(t')dt' = \begin{bmatrix} k_x(t) \\ k_y(t) \\ k_z(t) \end{bmatrix} = \begin{bmatrix} \gamma\int_0^t G_x(t')dt' \\ \gamma\int_0^t G_y(t')dt' \\ \gamma\int_0^t G_z(t')dt' \end{bmatrix}$$

k is also known as the « **wave vector** »

Effects of MF gradients on NMR signal

Signal coming from the whole sample

$$\hat{S}(t) = \int \hat{S}(\mathbf{r},t)d\mathbf{r} \propto \int M_T(\mathbf{r})\exp[i\phi(\mathbf{r},t)]d\mathbf{r}$$

Using the « k » formulation

$$\hat{S}(\boxed{t}) \propto \int M_T(\mathbf{r})\exp[i\mathbf{r}.\mathbf{k}(t)]d\mathbf{r}$$

$$\Downarrow$$

$$\hat{S}(\mathbf{k}) \propto \int M_T(\mathbf{r})\exp[i\mathbf{r}.\mathbf{k}]d\mathbf{r} = \mathrm{FT}^{-1}(M_T(\mathbf{r}))$$

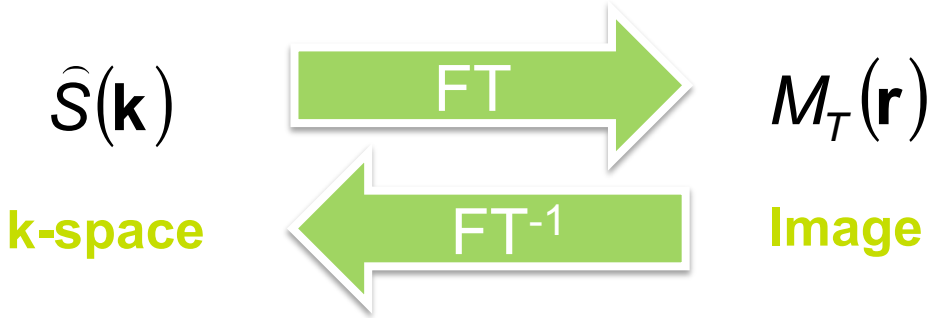
The k-space

Acquired NMR signal in the k-space

Time-domain NMR signal is represented as function of **k**
Reparameterization of acquired signal

$$\hat{S}(\mathbf{k}) \propto \mathrm{FT}^{-1}(M_T(\mathbf{r}))$$

$$\mathrm{FT}\left[\hat{S}(\mathbf{k})\right] \propto \mathrm{FT}\left[\mathrm{FT}^{-1}(M_T(\mathbf{r}))\right] = M_T(\mathbf{r})$$



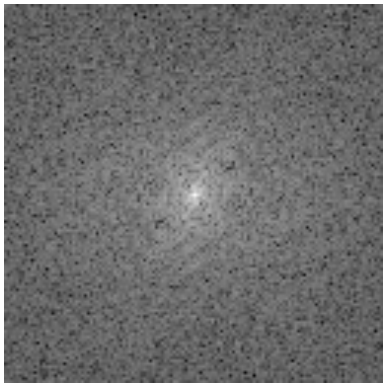
$\|\hat{S}\|(\mathbf{k})$

k-space

$M_T(\mathbf{r})$

Image

k_y

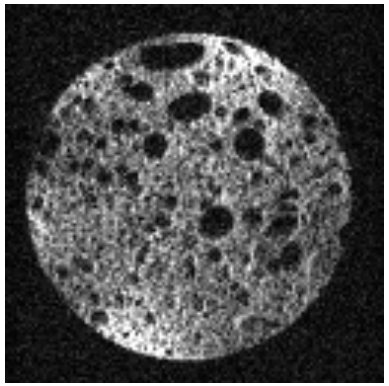


k_x

FT

FT^{-1}

y



x

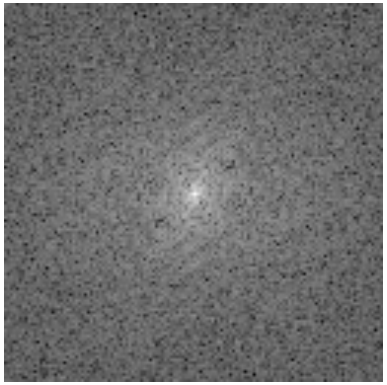
$\|\hat{S}\|(\mathbf{k})$

k-space

$M_T(\mathbf{r})$

Image

k_y

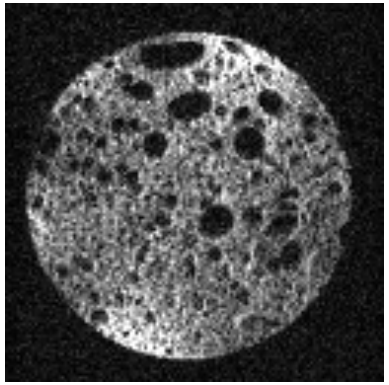


k_x

FT

FT^{-1}

y



x

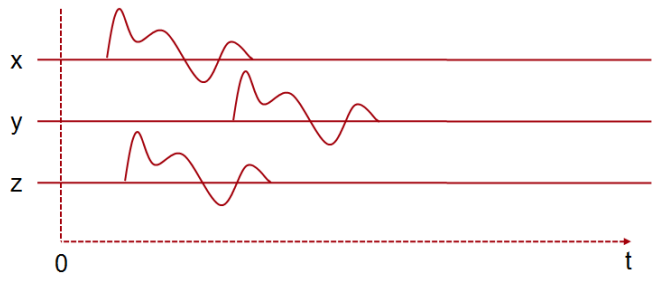
Remember … S is a complex number !

z is orthogonal to the slide !

How riding about the k-space ?

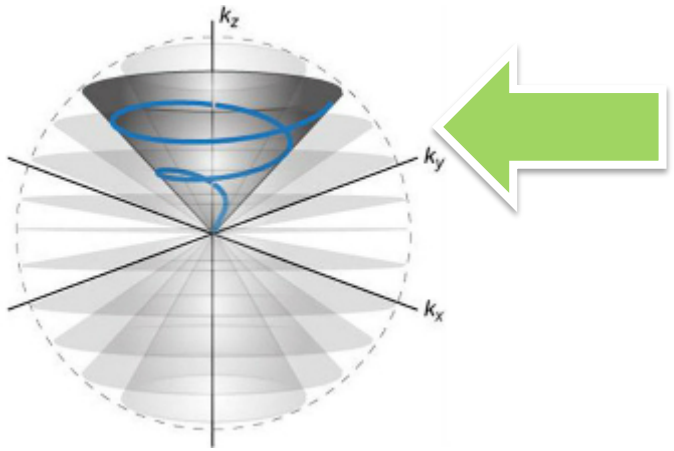
(1) Change the gradients with time

$$\mathbf{G}(t) = \begin{bmatrix} G_x(t) \\ G_y(t) \\ G_z(t) \end{bmatrix}$$



(2) Calculate the wave-vector = integration

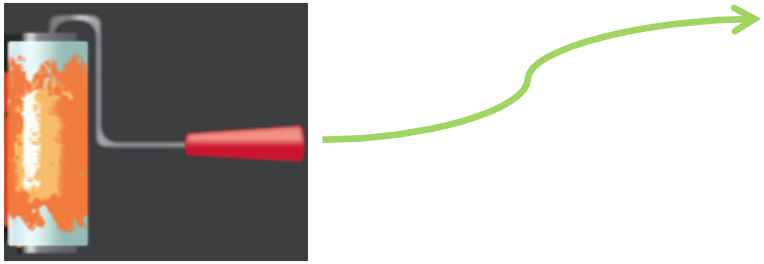
$$\mathbf{k}(t) = \gamma \int_0^t \mathbf{G}(t')dt'$$



3D trajectory

How filling the k-space ?

(1) **Change the gradients =** displacement along the 3D trajectory

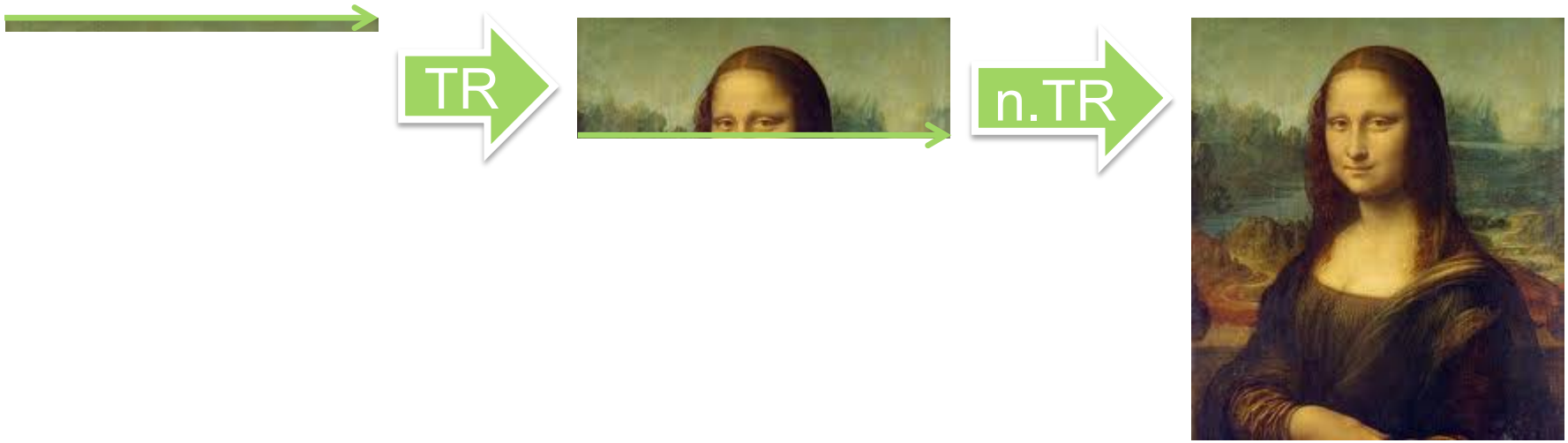


(2) **Acquire the signal =** fill the k-space with the (complex) value of NMR signal

How filling the k-space ?

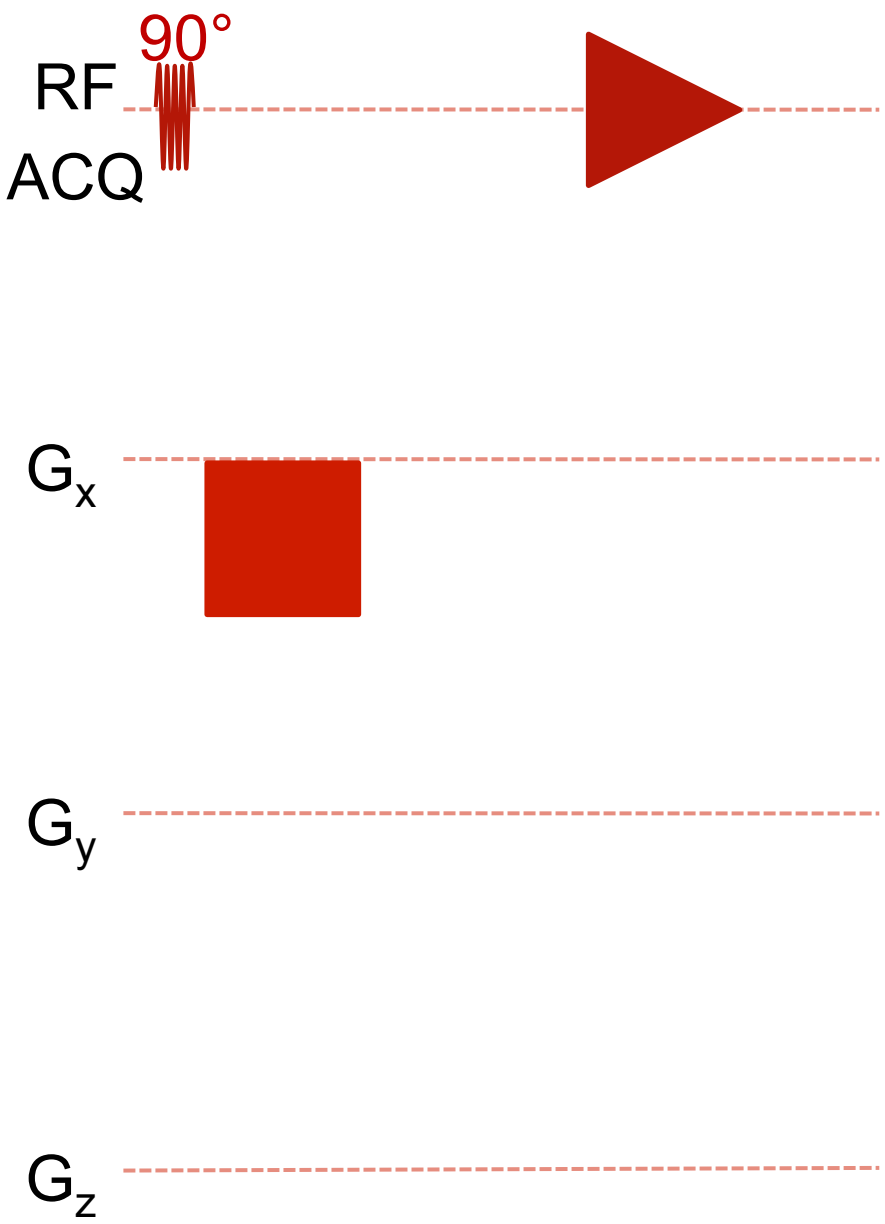
Many ways

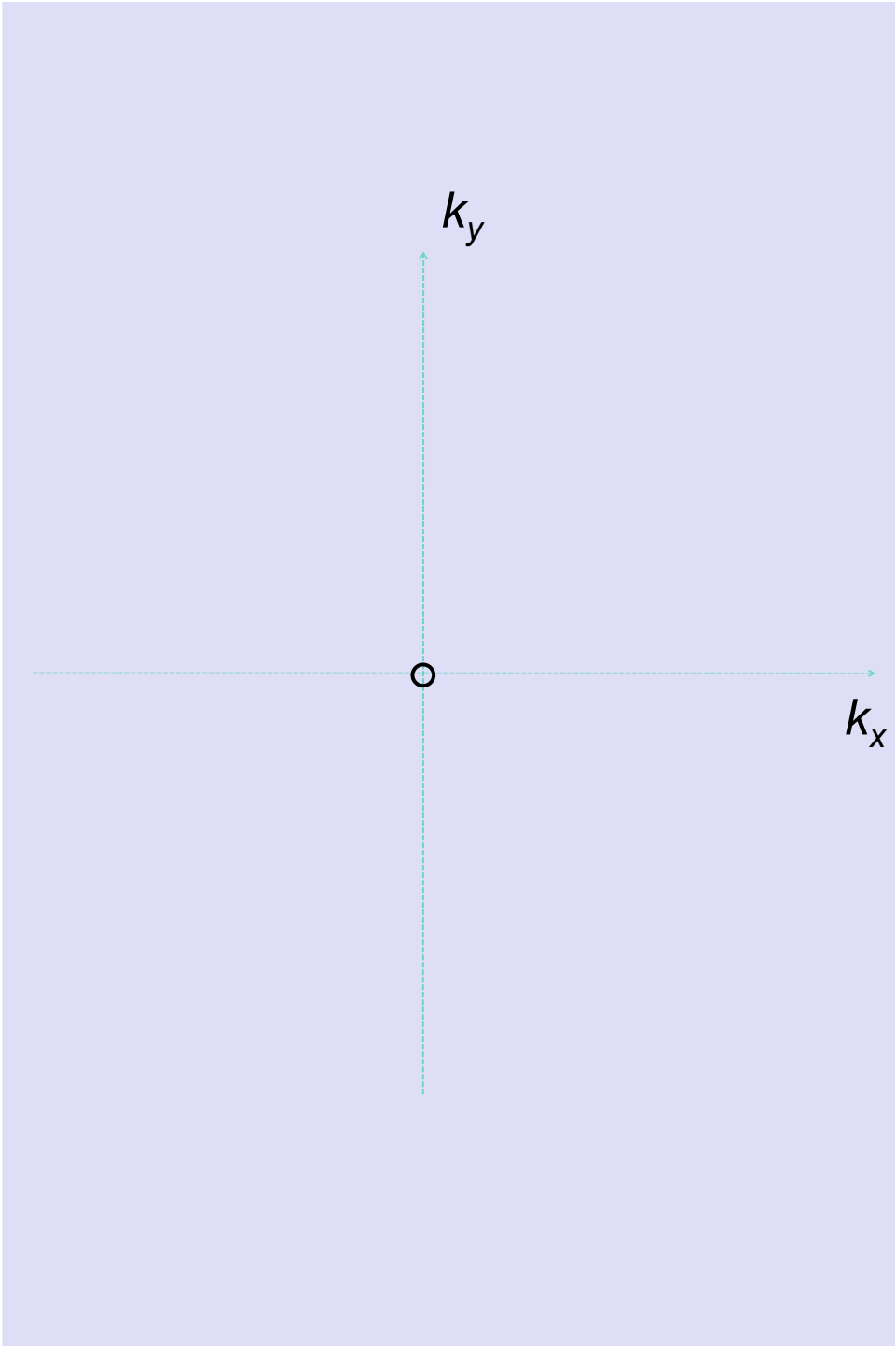
Multi-steps / Cartesian

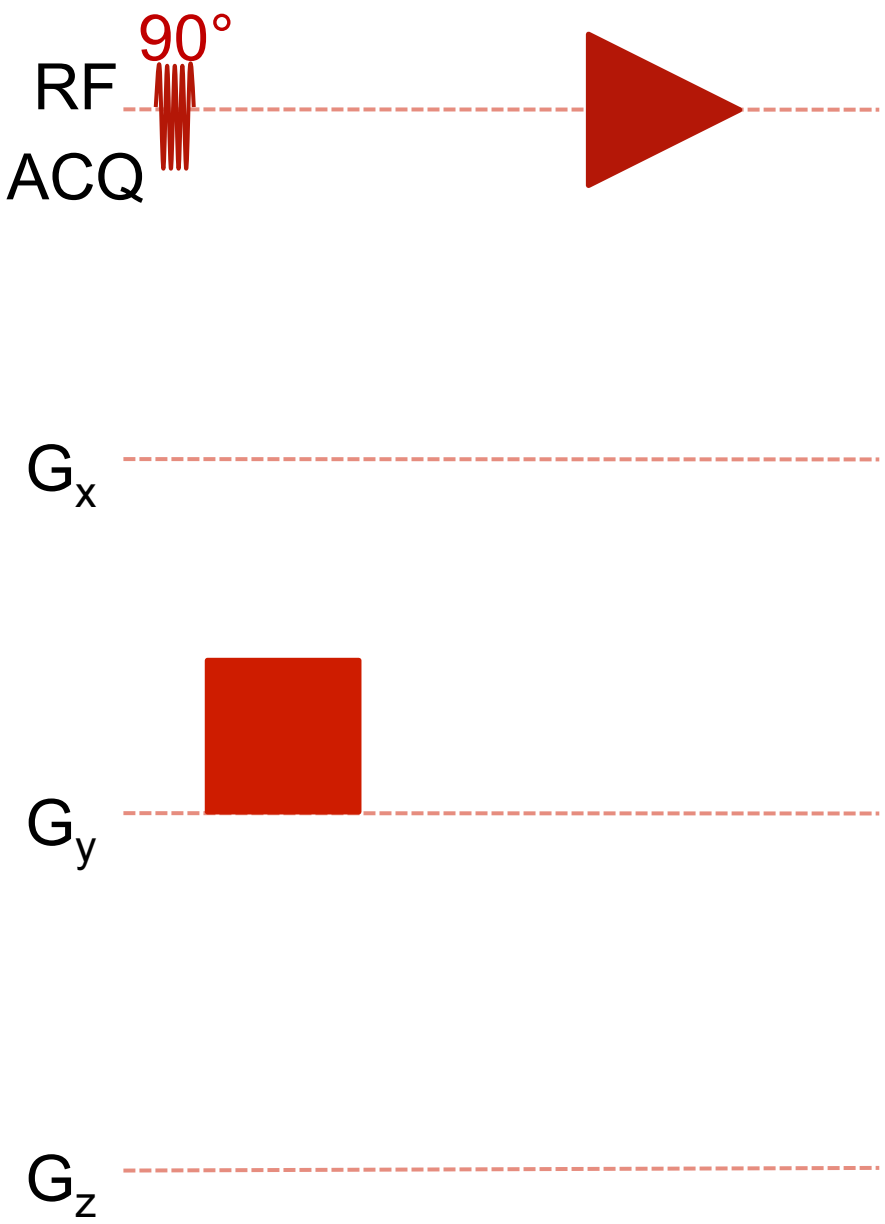


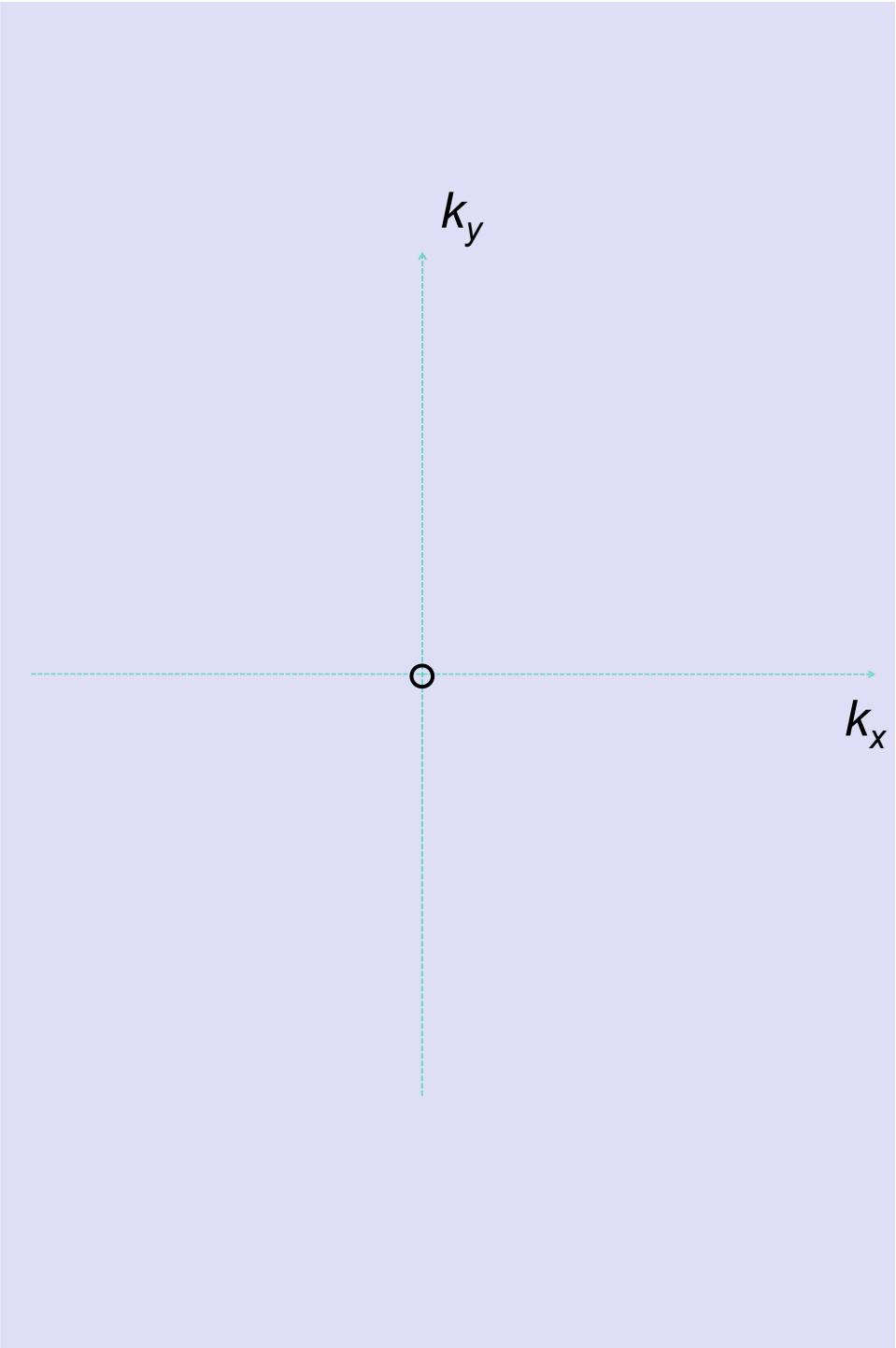
… and many others !

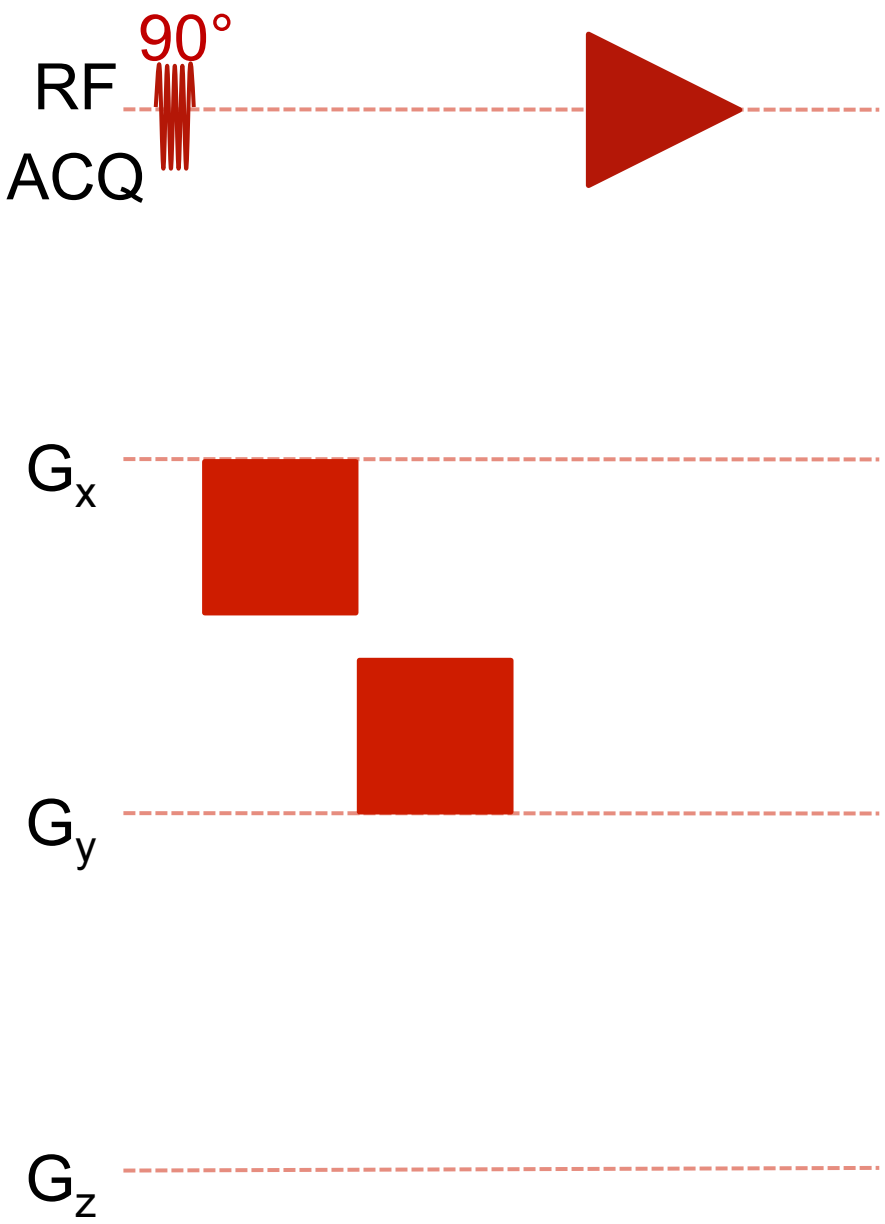
Pure « phase » encoding

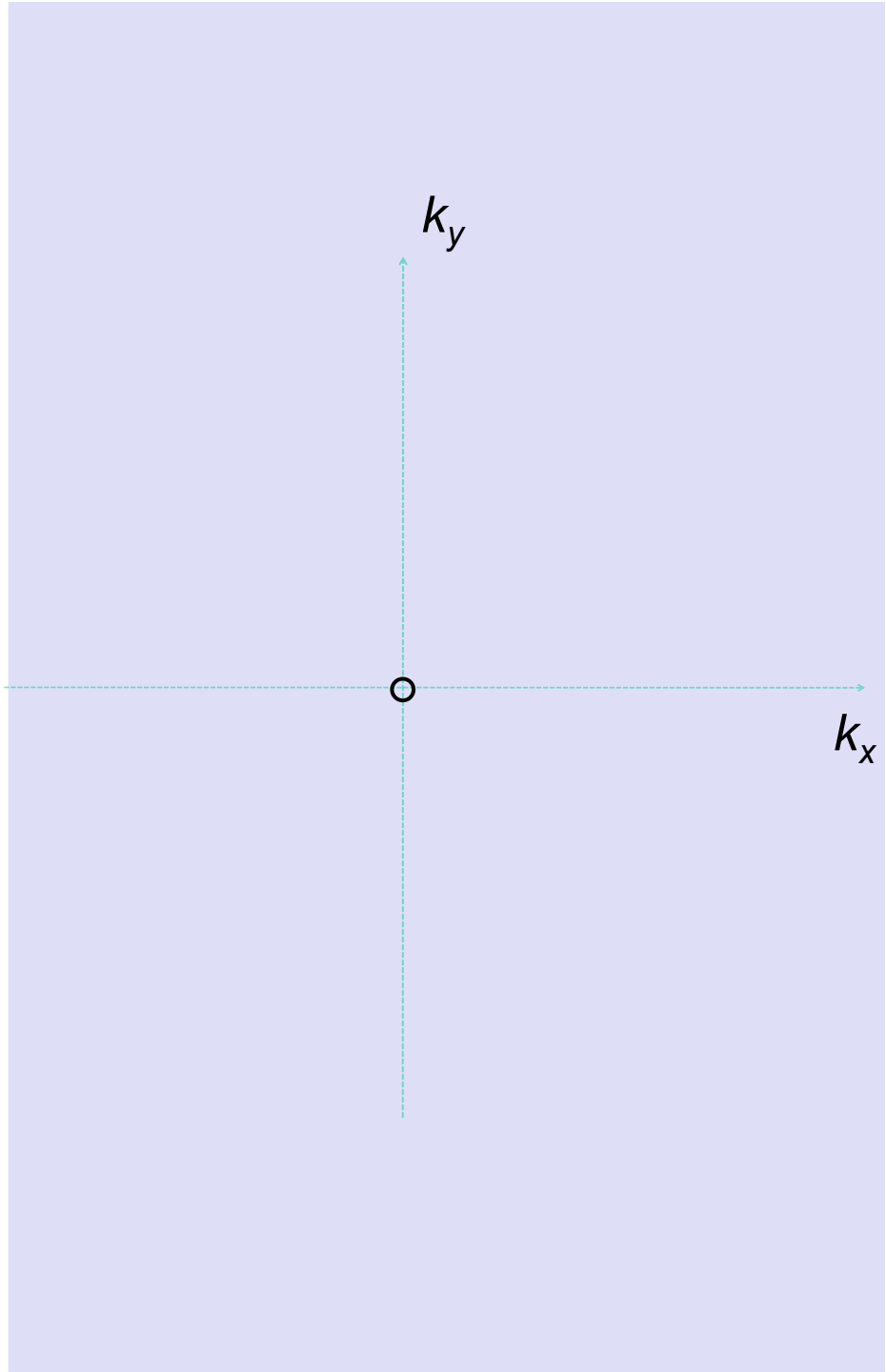

90°
RF
ACQ
G_x
G_y
G_z

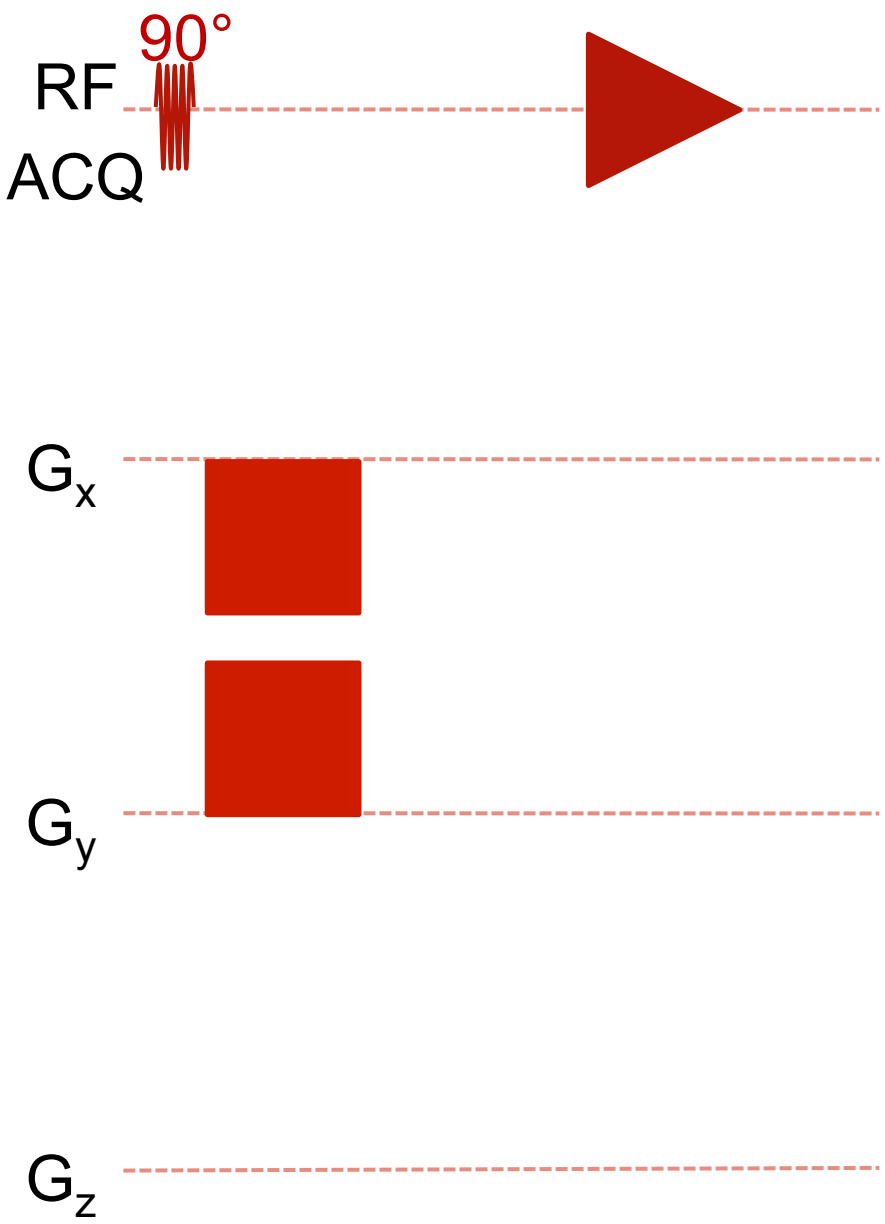

k_y
k_x

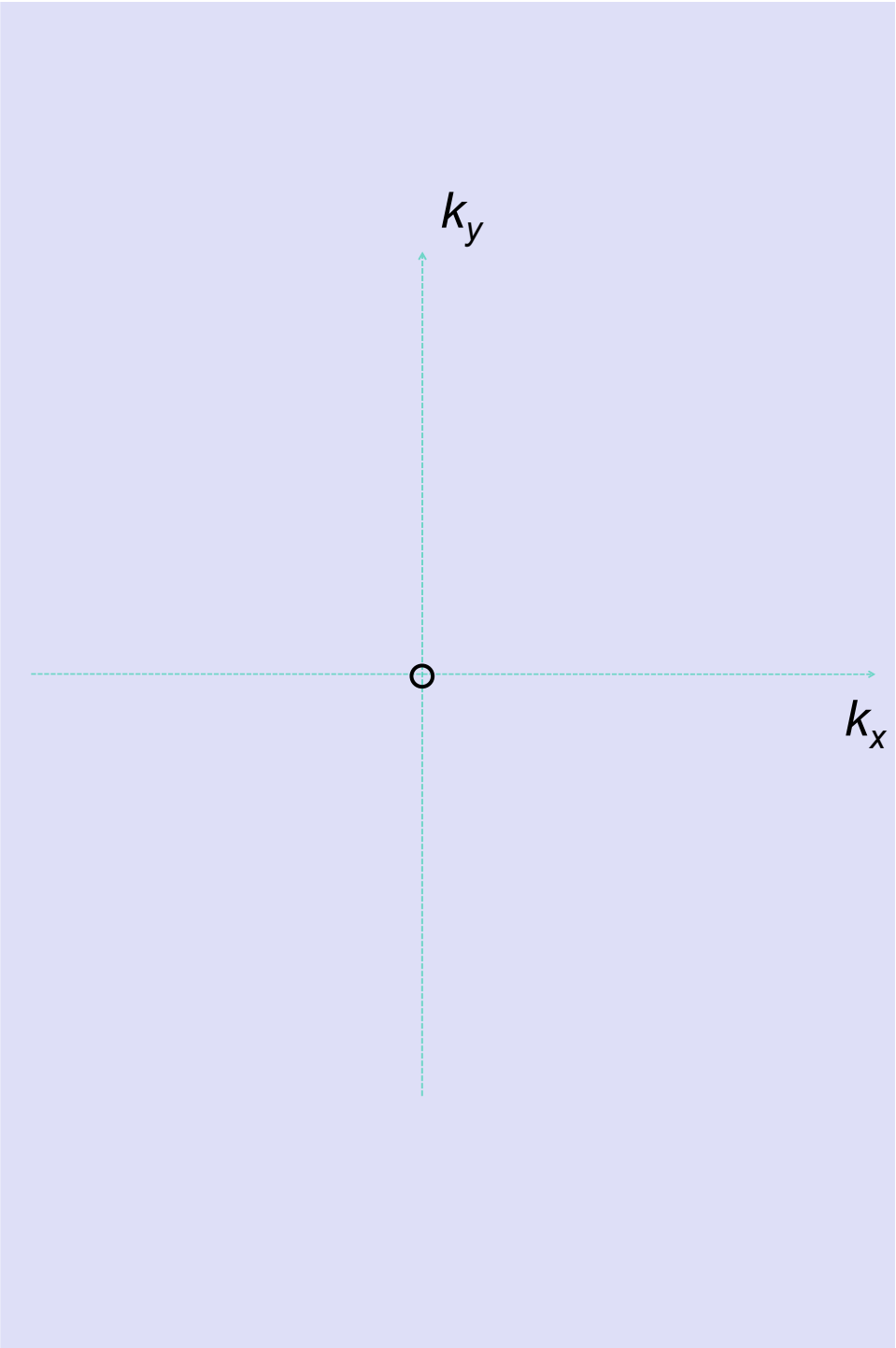

90°
RF
ACQ
G_x
G_y
G_z

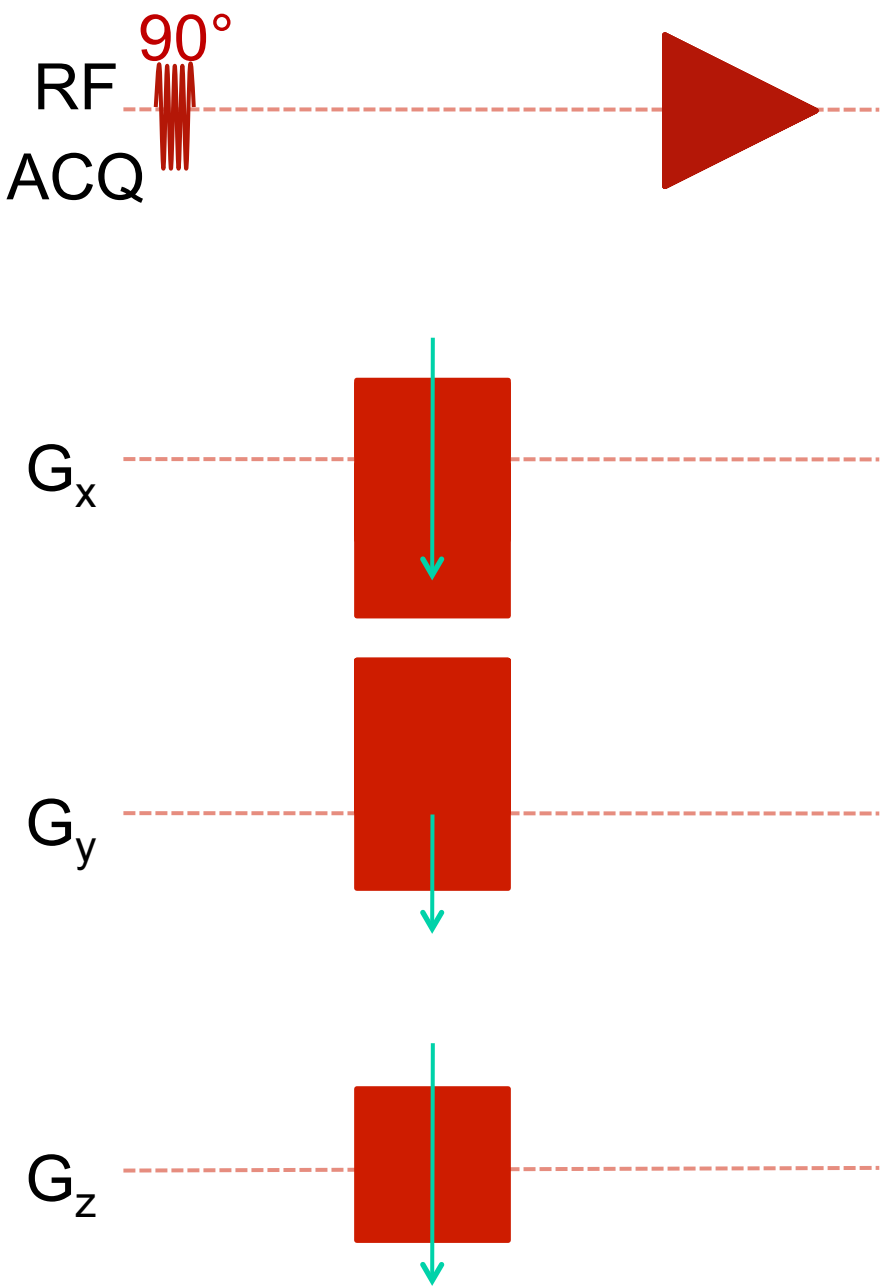

k_y
k_x

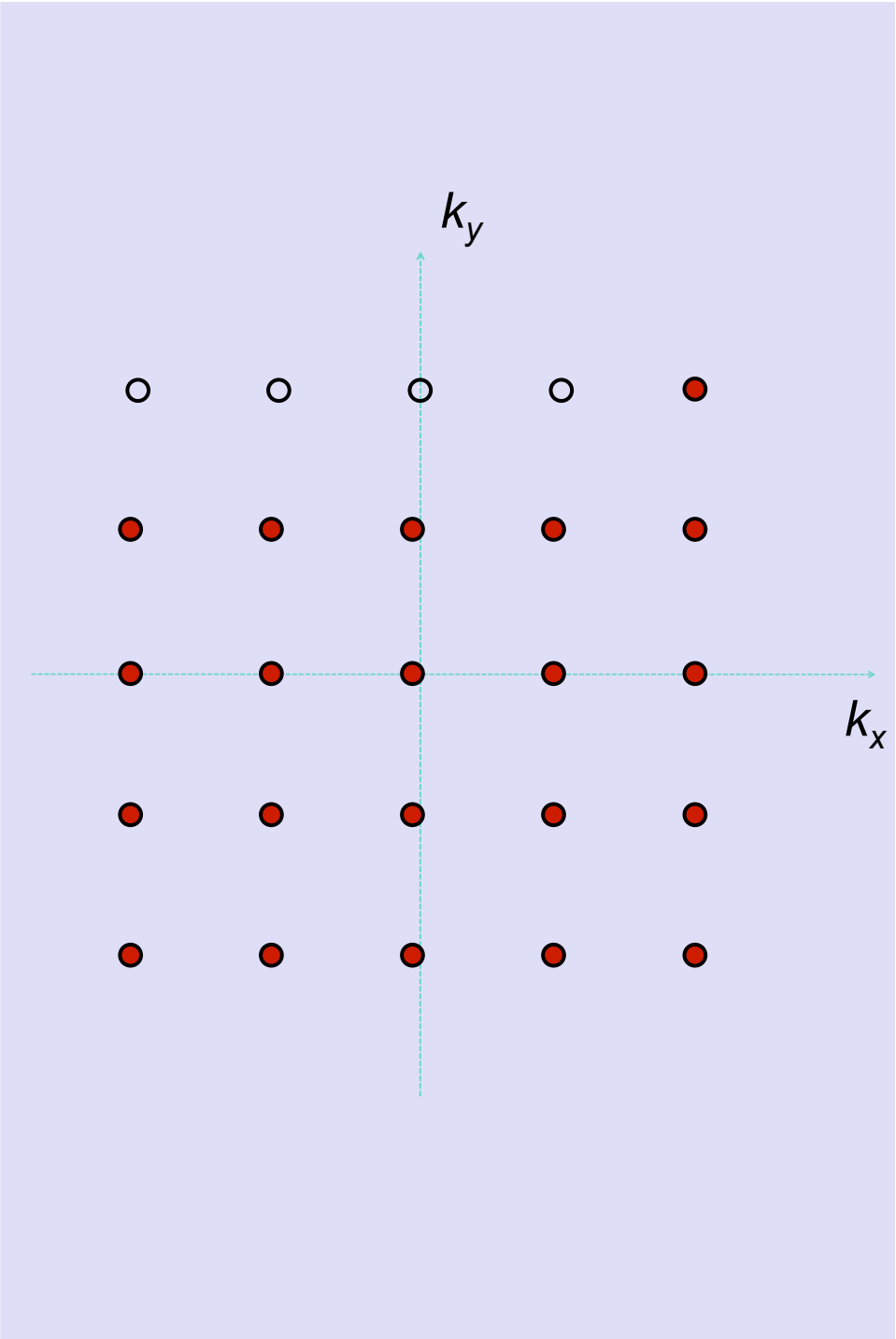

90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x

Pure phase encoded imaging

Cartesian trajectory over the k-space

Gradients before the acquisition

« Phase » encoding gradient pulses
Encode the 3 directions

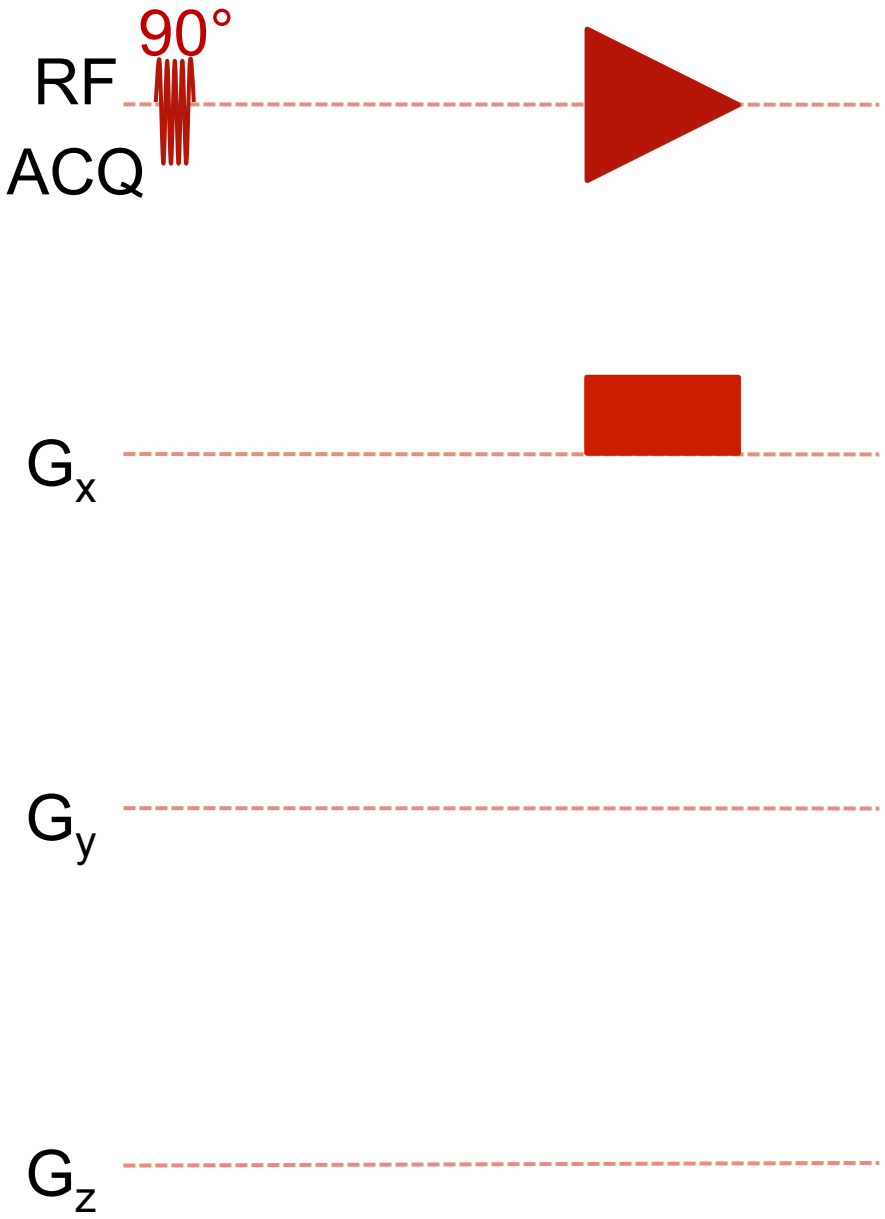
Whole FID available

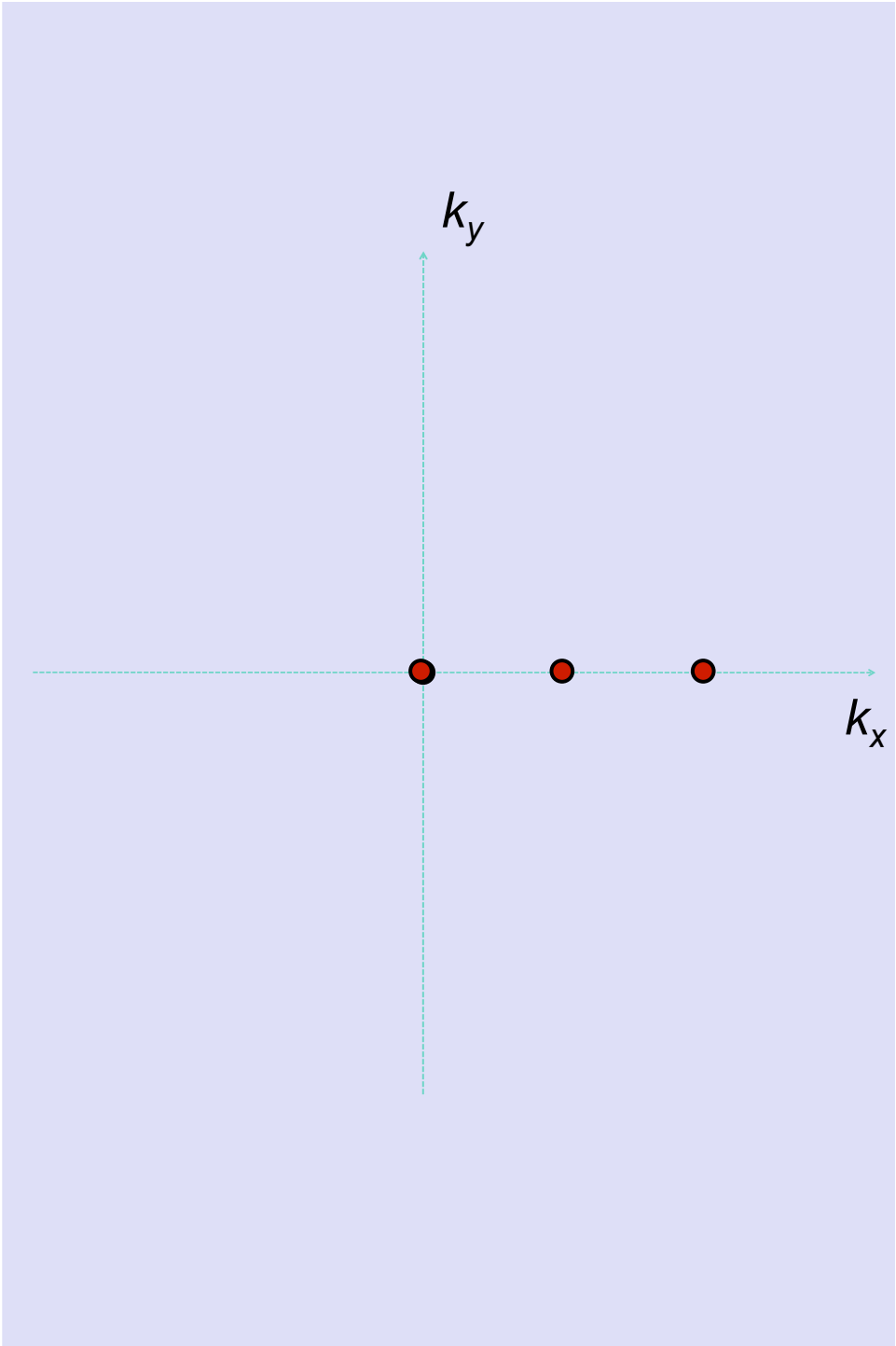
All the chemical shift information is available !

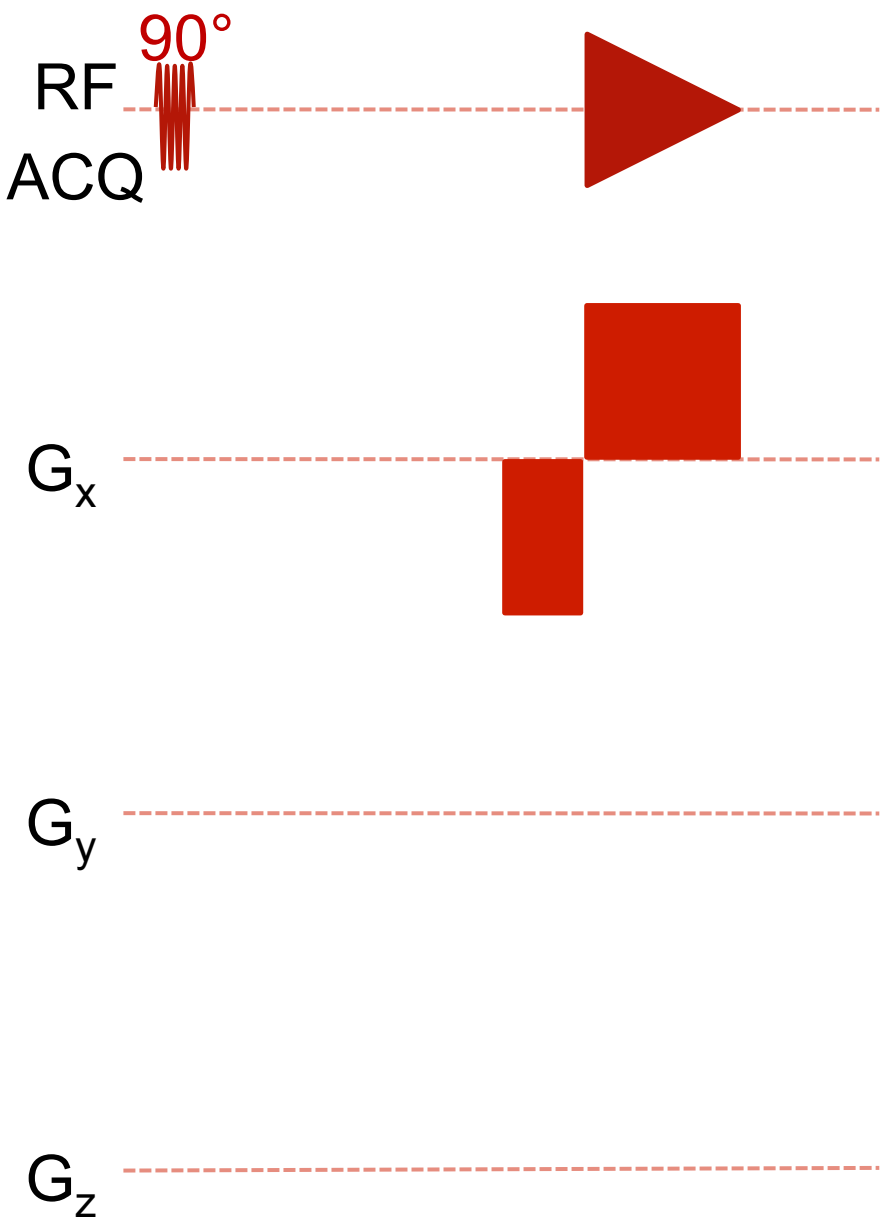
Acquisition time

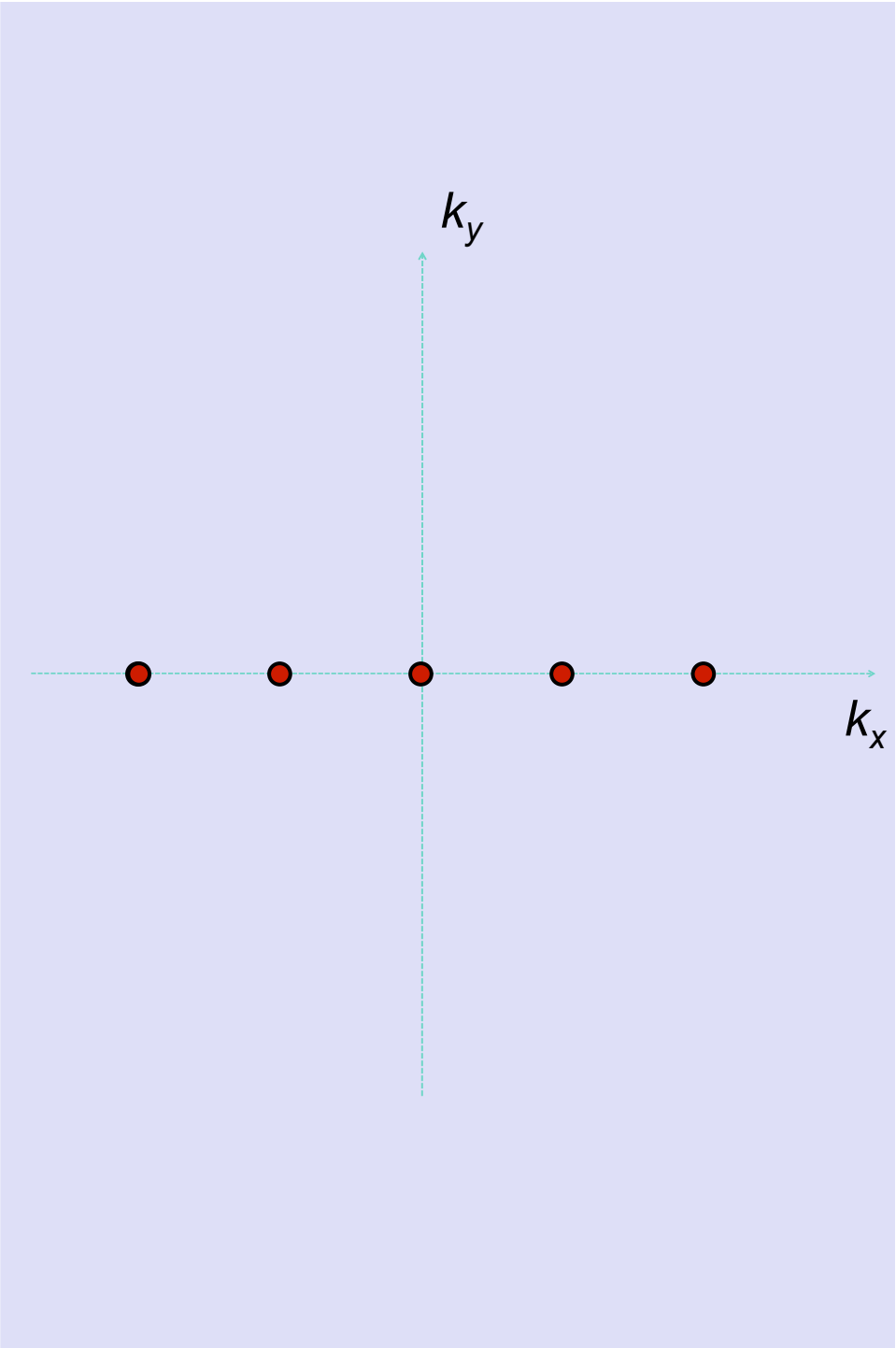
TA = Nx Ny Nz TR
= 64.64.64. 1s = 73h

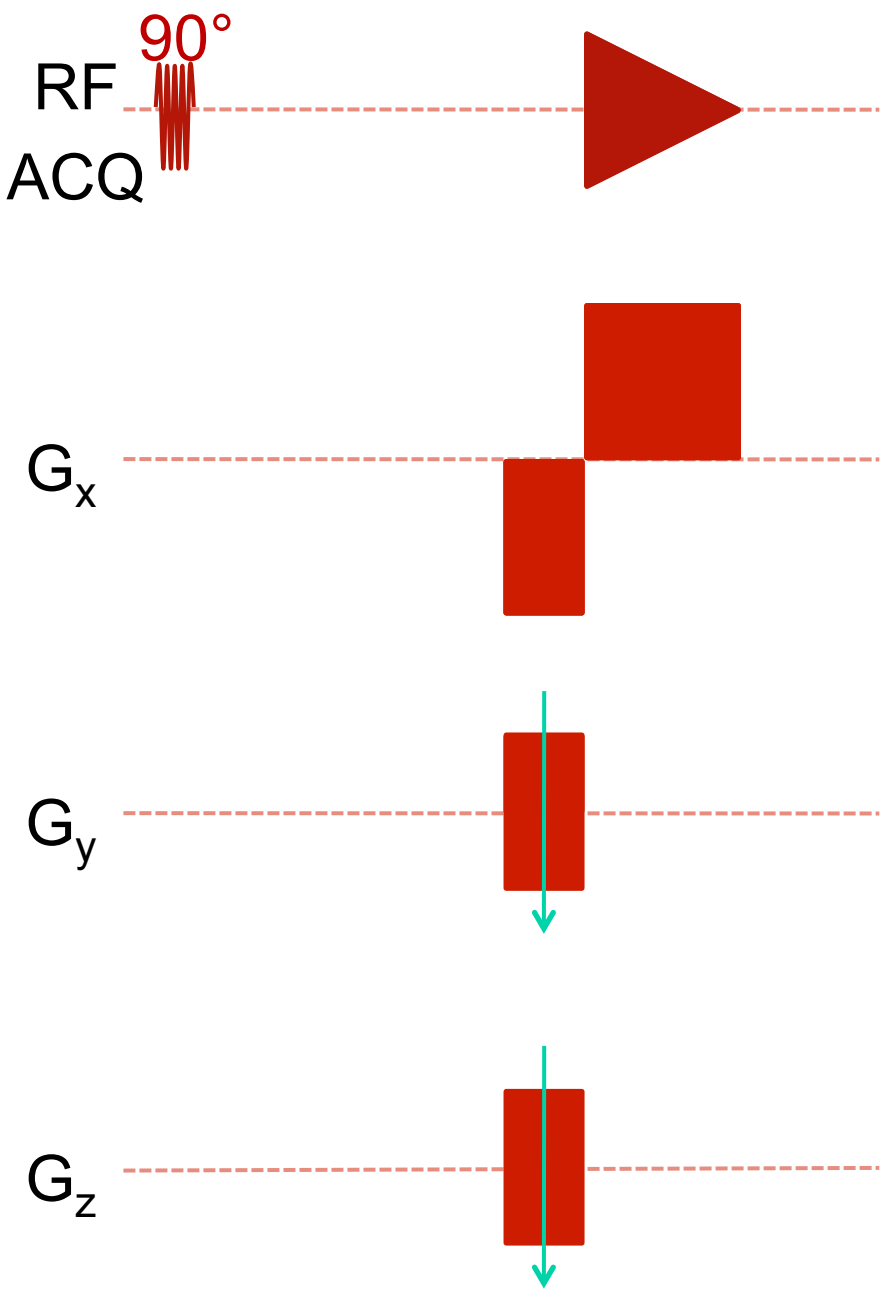
Spin Warp

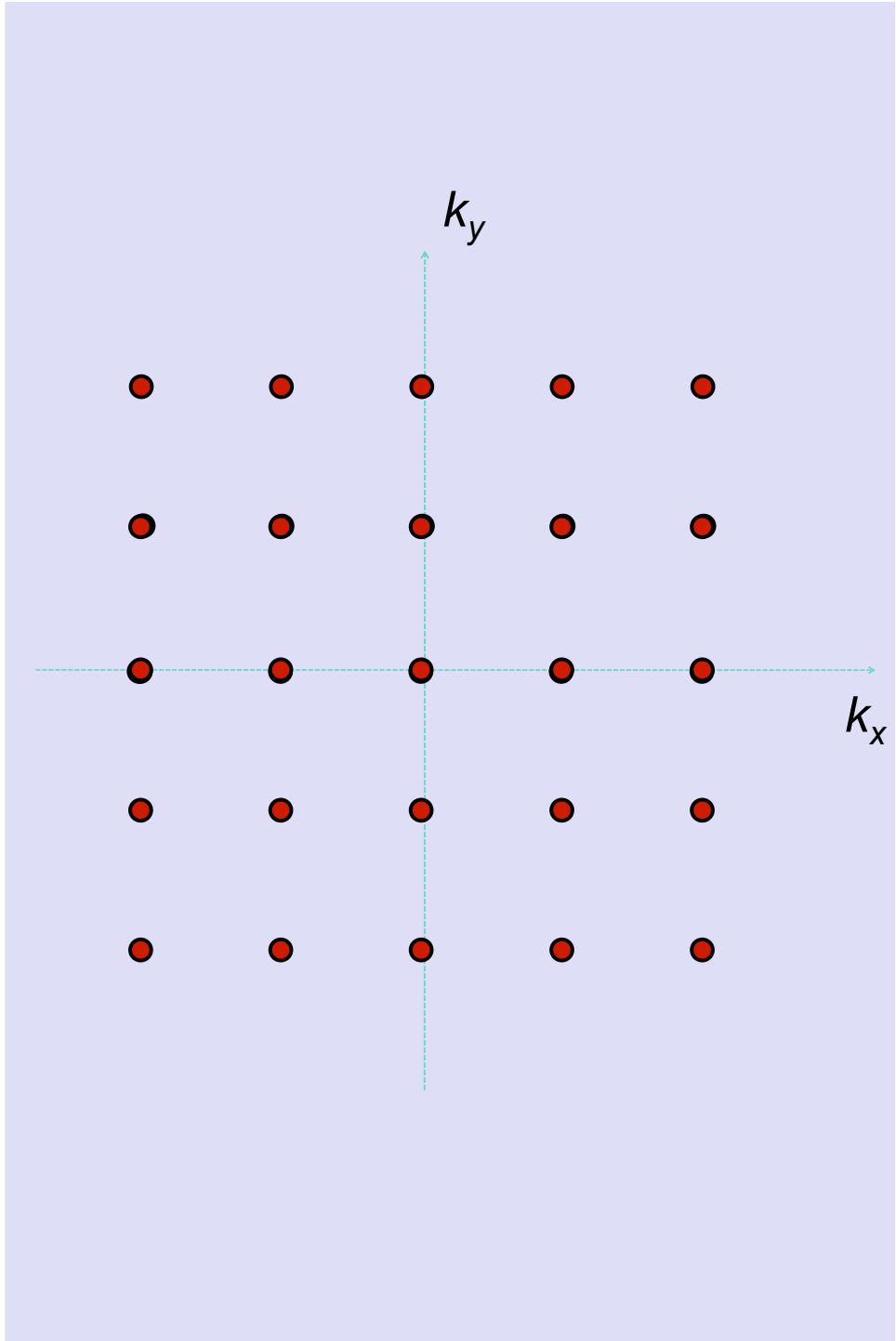

90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x

Spin warp

Cartesian trajectory over the k-space

Gradients during the acquisition

« Read » gradient pulses

Encode one direction

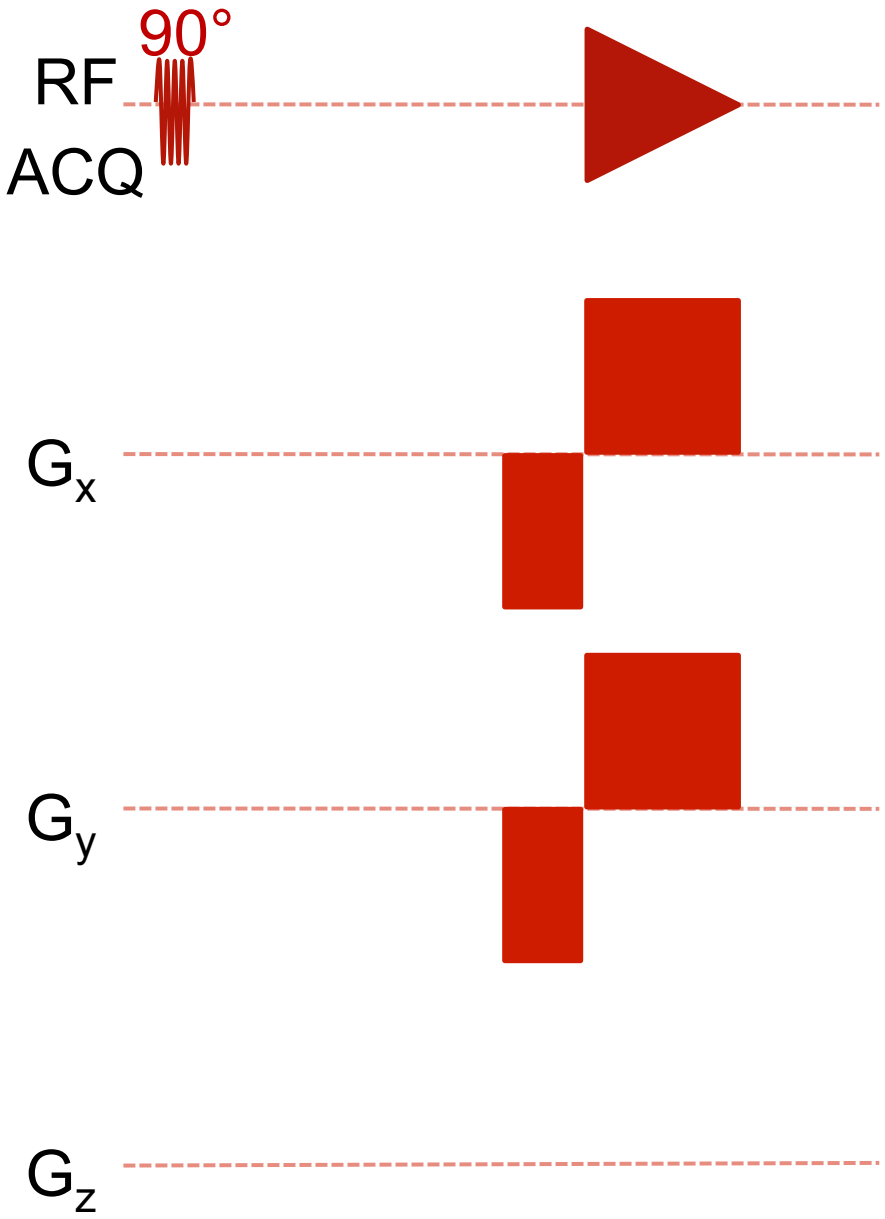
« Phase » encoding in the 2 other directions

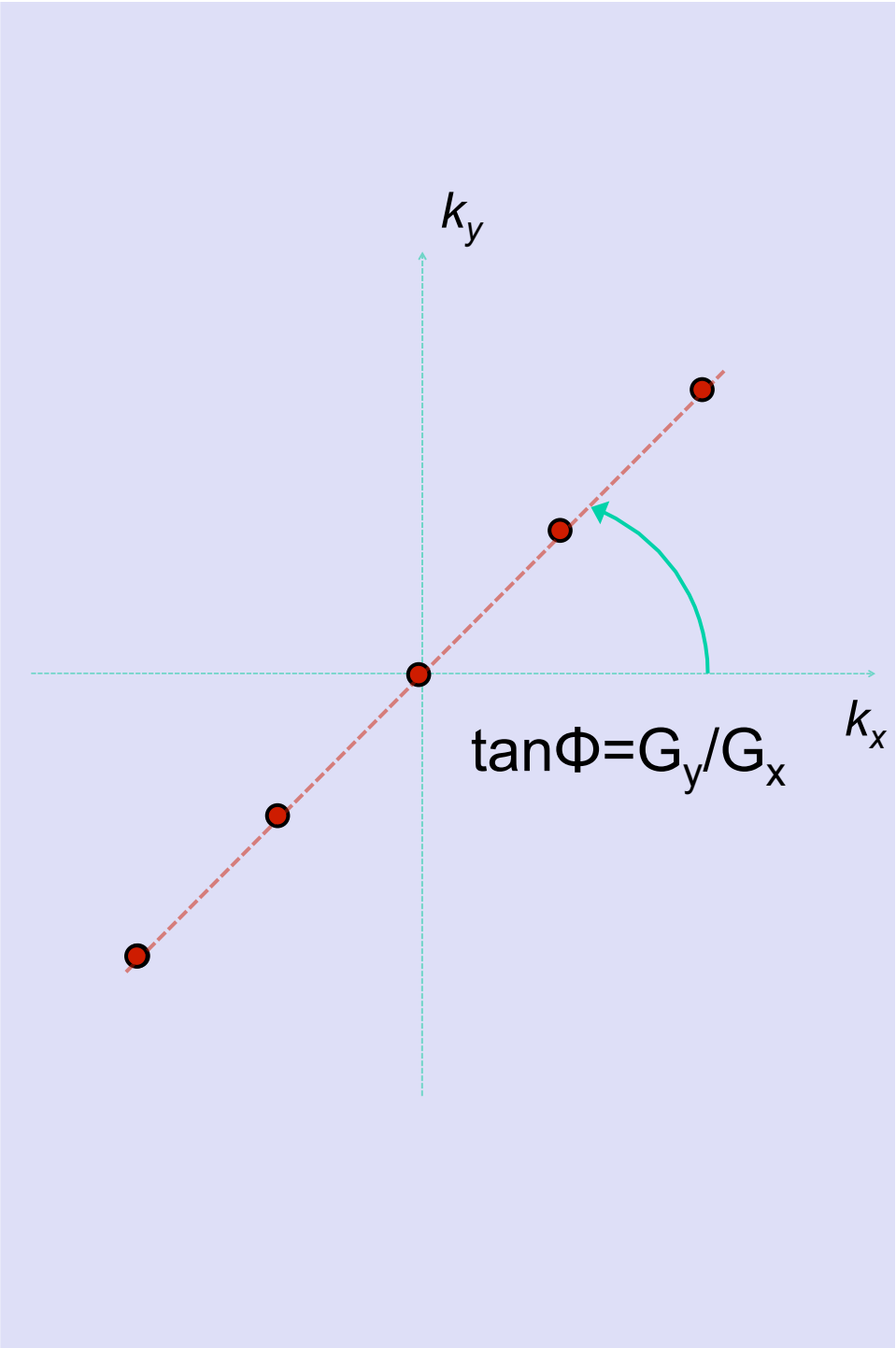
Acquisition time

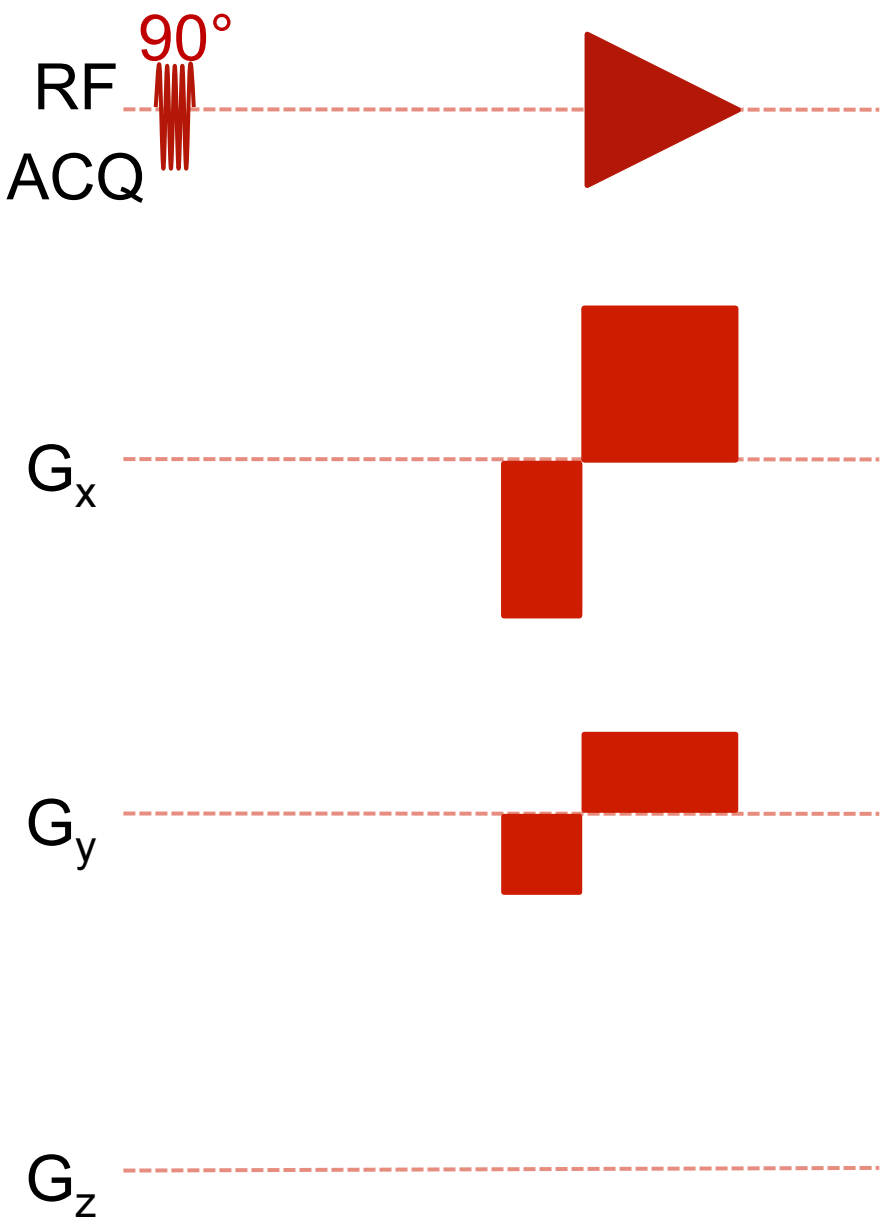
TA = Nx Ny Nz TR

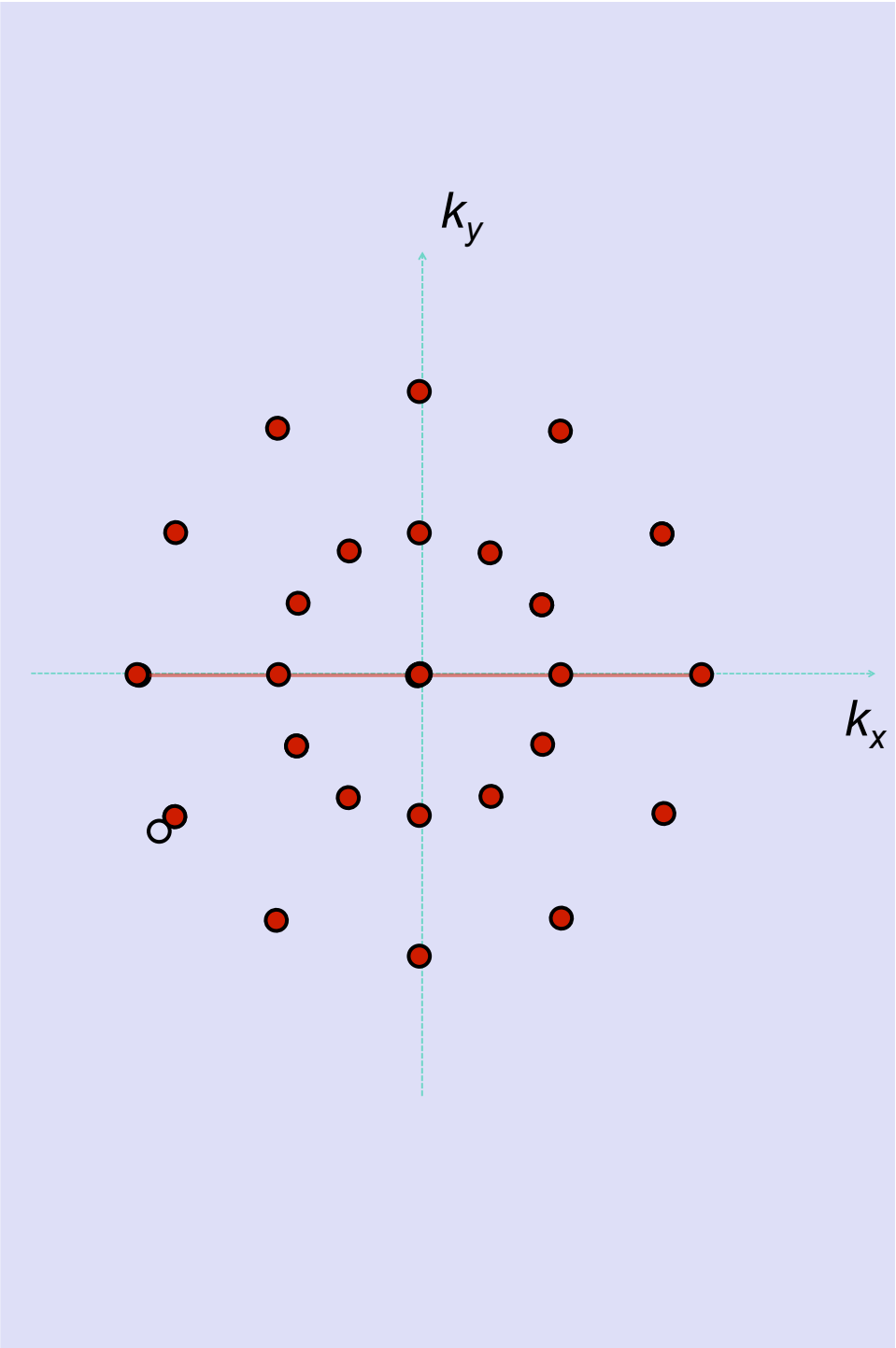
= 64.64. 1s = 1h8’

Radial encoding

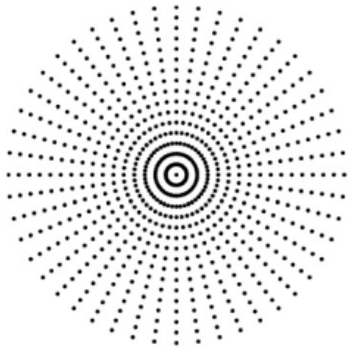

90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x
$\tan\Phi = G_y/G_x$


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x

Radial encoding



Radial trajectory over the k-space

Non-uniform density

Gradients during the acquisition

Only read-gradient pulses

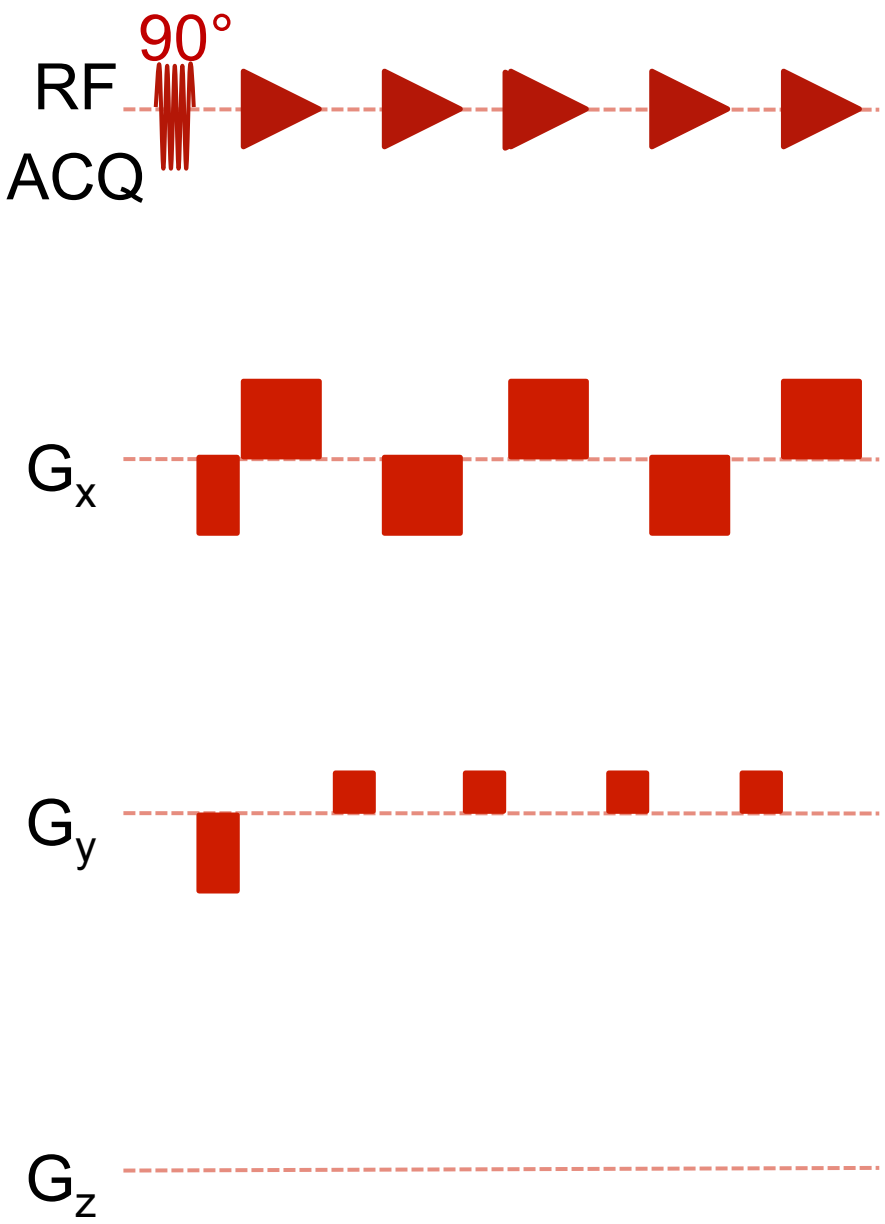
No phase encoding

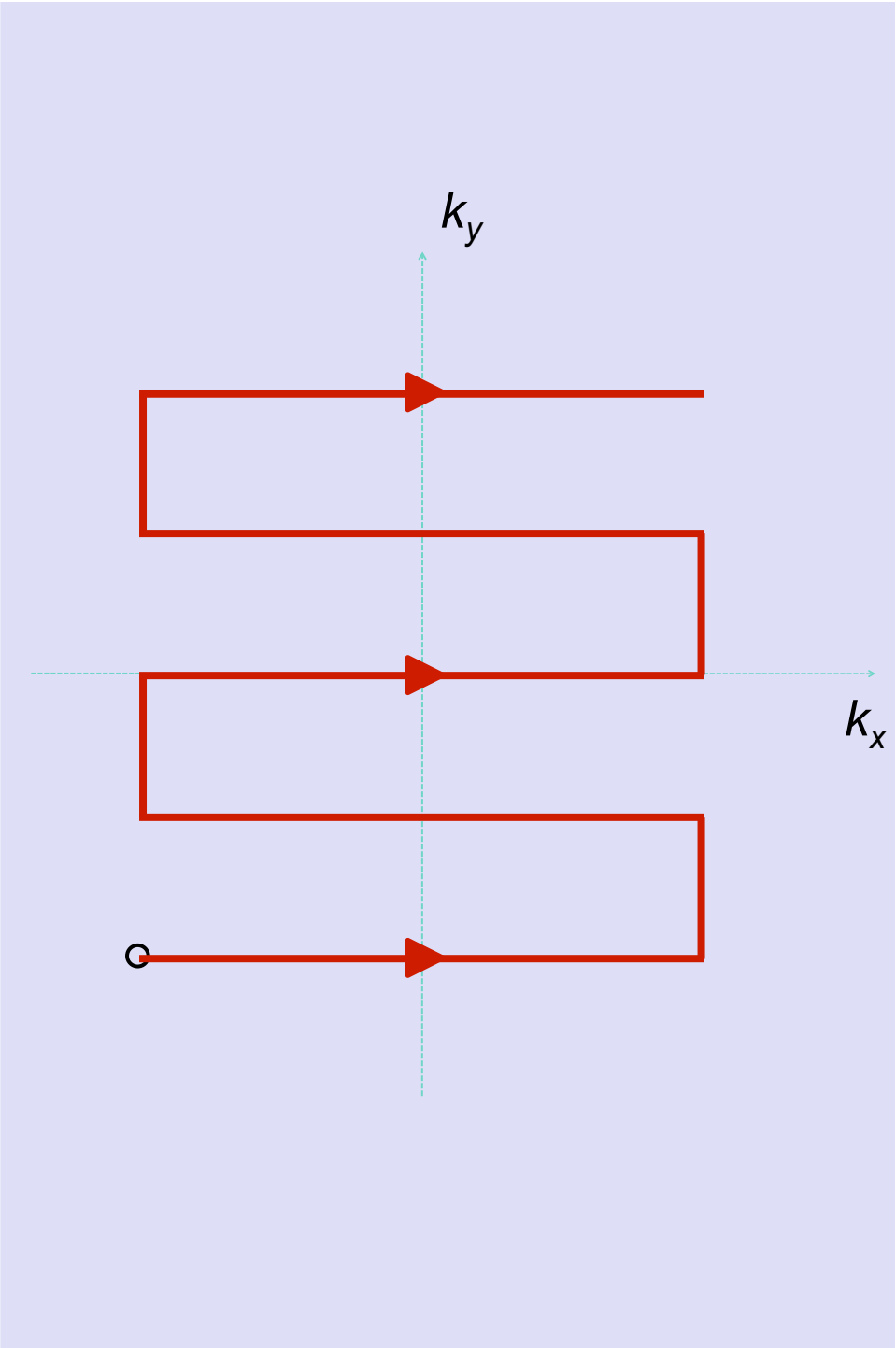
Acquisition time

TA = $N\phi\ N\theta\ TR$

= 64.64. 1s = 1h8’

Echo planar


90°
RF
ACQ
G_x
G_y
G_z


k_y
k_x

Echo planar

(Quasi) Cartesian trajectory over the k-space

Alternated read gradients during the acquisition

Short phase encoding in-between the read gradients

Blip

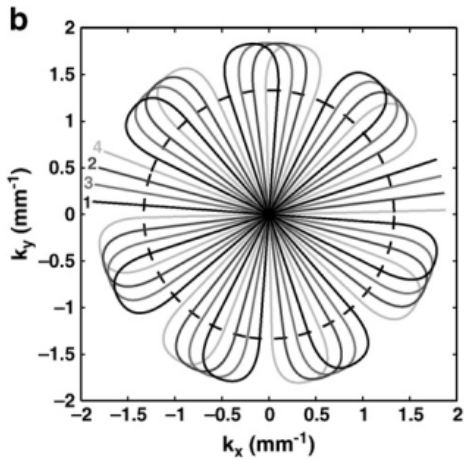
Acquisition time

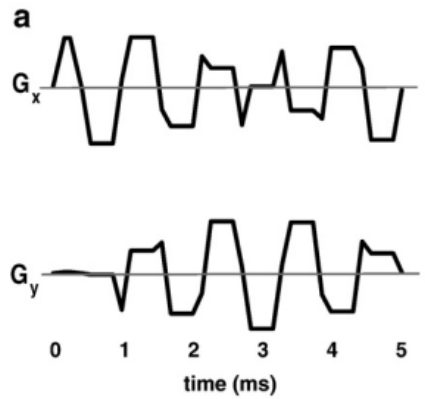
TA = TR

= 1s

Design of any trajectories

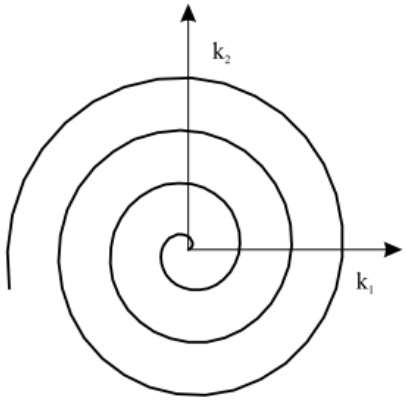
$$\mathbf{k}(t) = \gamma \int_0^t \mathbf{G}(t')dt' \quad \Rightarrow \quad \mathbf{G}(t') = \frac{\partial \mathbf{k}(t)}{\gamma \partial t}$$

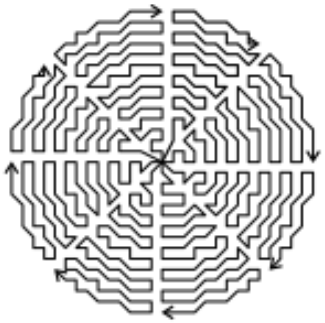




Lee, *Neuroimage* (2010)

« Exotic » trajectories





Khrapitchev, *J Magn Reson* (2006)

(Some) Limits

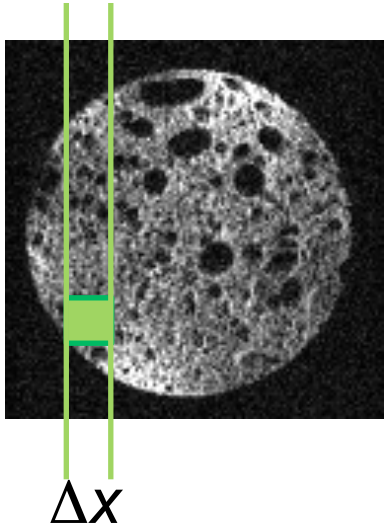
Spatial resolution

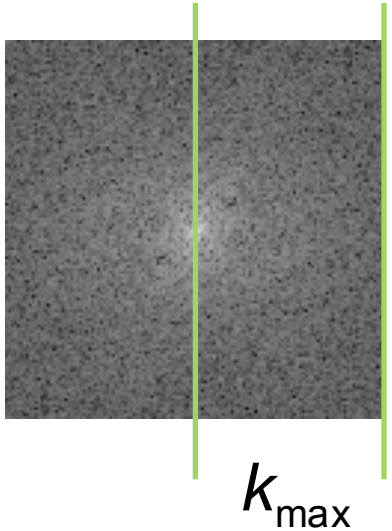
Signal-to-noise ratio

V = voxel volume

$$SNR \propto V = \Delta x.\Delta y.\Delta z$$

Voxel size

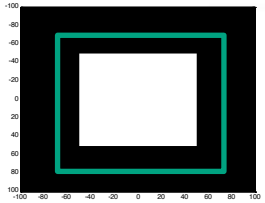




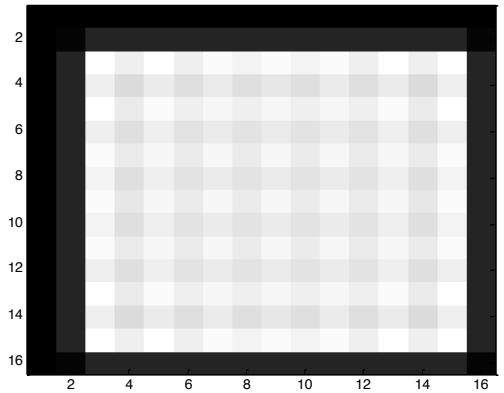
$$\Delta x \propto k_{max}^{-1}$$

High resolution = high k_{max} = Strong gradients

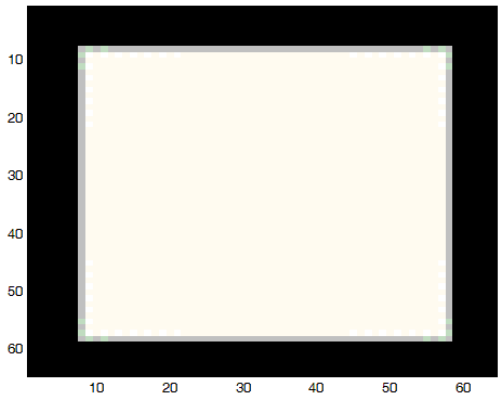
Sampling



Object



Spin warp encoding
FOV = 128 mm
16 x 16



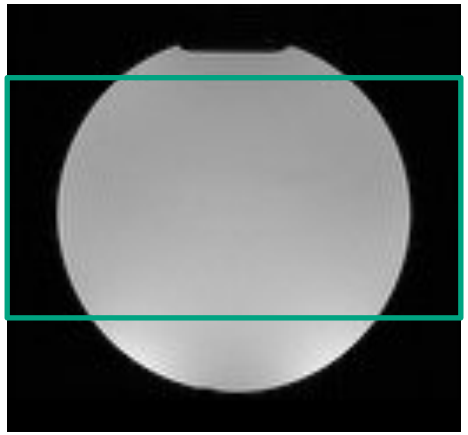
Spin warp encoding
FOV = 128 mm
64 x 64

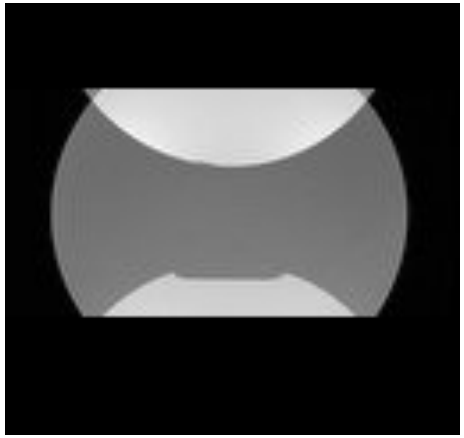
Aliasing

Field of view (FOV) should contain the object

Solutions exist for reducing the object size

- Saturation band
- Selective pulse
- Surface coil





Off-resonance / Gradient imperfections

Off-resonance

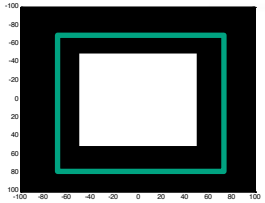
Frequency shift $\Omega(\mathbf{r})$

Trajectory perturbation
Differences between experimental and theoretical gradient waveforms

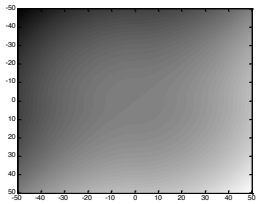
$$\hat{S}(\mathbf{k}(t)) \propto \int M_T(\mathbf{r}) \exp\left[i\left(\mathbf{r}.\boxed{\mathbf{k}^*(t)} + \boxed{2\pi\Omega(\mathbf{r})t}\right)\right] d\mathbf{r}$$

Trajectory perturbation
All the off-resonance perturbations contribute

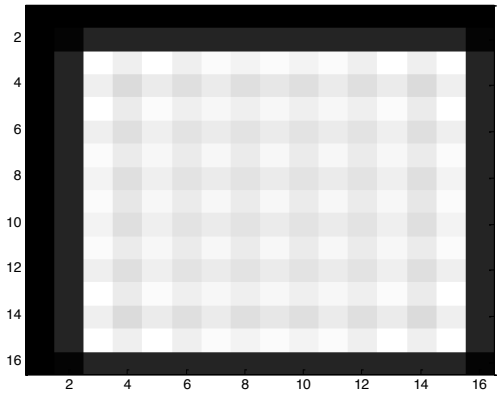
Off-resonance



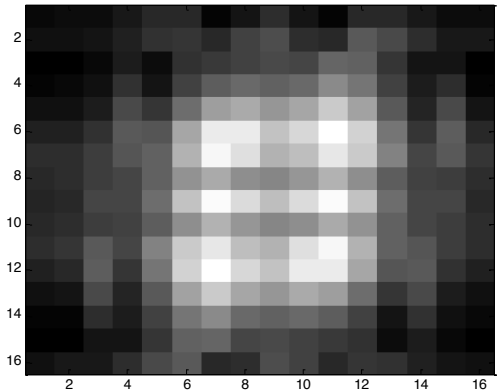
Object



Off-resonance



Spin warp encoding
FOV = 128 mm
16 x 16
No perturbations



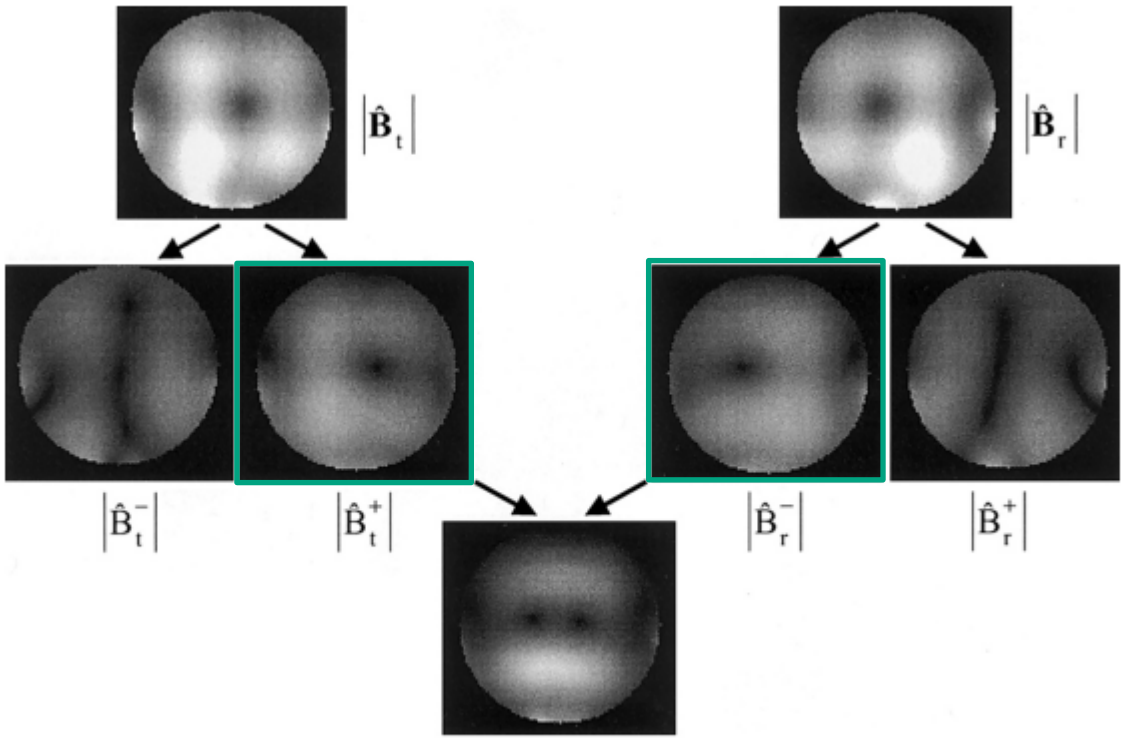
With perturbations

Inhomogeneities of B1 fields

Quadrature surface coil

7 T

Transmission



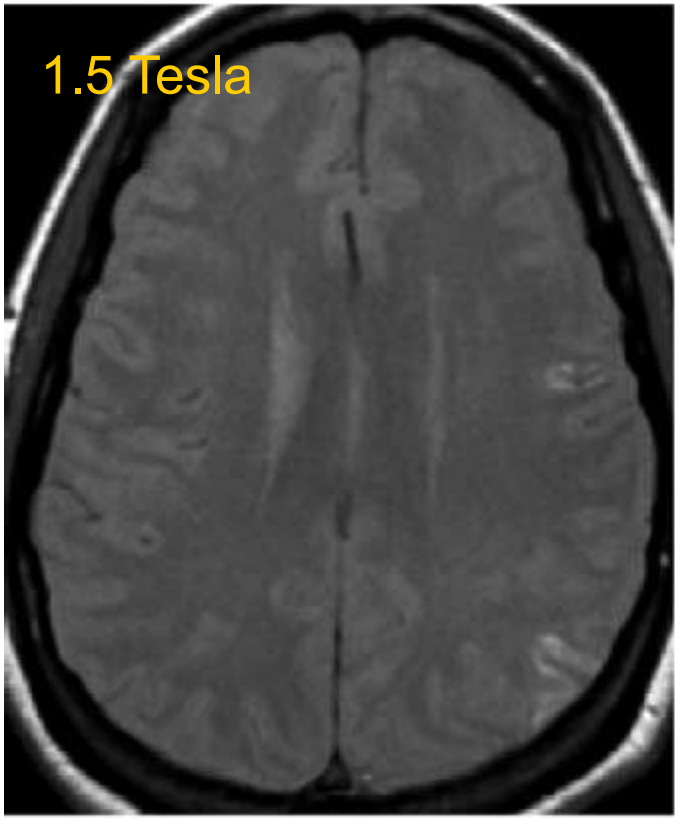
Reception

Resulting image

Homogeneous phantom

Wang *et al*, Magn Reson Med (2002) 48:362 - 369

Inhomogeneities of B1 fields



Novak *et al*., Magn. Reson. Imaging (2005)

Selective excitation

For reducing the number of directions to encode

Band-limited pulse
Pulse applied with a gradient pulse