Spatial encoding in Magnetic Resonance Imaging

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What’s an image?

« … a reproduction of a material object by a camera or a related technique »

Multi-dimensional signal

\[ I(r) \quad r = \text{vector of spatial coordinates} \quad (x,y,z)^T \]

Signal intensity of the image

\[ I \quad \text{is scalar} = \text{Image with a single frame} \]

\[ I \quad \text{is a vector} = \text{Multi-channel image} \]
The first MR image
Made by Paul LAUTERBUR, Nature (1973)

Fig. 1 Relationship between a three-dimensional object, its two-dimensional projection along the Y-axis, and four one-dimensional projections at 45° intervals in the XZ-plane. The arrows indicate the gradient directions.

Fig. 2 Proton nuclear magnetic resonance tomogram of the object described in the text, using four relative orientations of object and gradients as diagrammed in Fig. 1.
There are 3 spatial dimensions

In *real* world, images should be in 3D

Human brain – $V = 3 \times 3 \times 3 \text{ mm}^3$
1D or 2D images
The concept of « profile »

\[ I(x) = \int I(x, y, z) dydz \]

Multi-channel images?

MRI provides different contrasts

Clark et al, J Sci Food Agric (1998) 78
The MRI challenges

More sensitivity

Improvement of spatial and temporal resolutions

More specificity

Tissue discrimination through the signature given by the signal I
How to encode spatial information?

The target

Amplitude $M_T$ of transverse magnetization over space

$$M_T(r)$$

$r$: Spatial Cartesian coordinates

$$r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Assumptions

- No relaxation
- Uncoupled spins
Qu'est-ce qu'une image ?

Signal without encoding

Free induction decay

RF / ACQ

Flip angle

$\text{t}$
Signal without encoding

FID

\[ \hat{S}(r, t) = M_T(r) \exp[i\phi(r, t)] \]

\( \hat{S} \) Complex signal because of quadrature detection

\[ \phi(r, t) = 2\pi \int_0^t [f(r, t') - f_0] dt' \]

\( \phi \) Phase accumulated during \( t \) period

On-resonance signal coming from the whole sample

\[ \hat{S} = \int \hat{S}(r, t) dr = \int M_T(r) dr \]
Quadrature detection

Components of $M_T$ in the rotating frame

Introduce phase/off-resonance information

$\text{Re}(\tilde{S})$ $\text{Im}(\tilde{S})$

What is a gradient?

**Mathematical operator**

**Vector** of partial derivatives with respect to each coordinate

For a 3D function \( \text{fun}(\mathbf{r}) = \text{fun}(x, y, z) \)

\[
\nabla \text{fun}(\mathbf{r}) = \left[ \begin{array}{c}
\frac{\partial \text{fun}(\mathbf{r})}{\partial x} \\
\frac{\partial \text{fun}(\mathbf{r})}{\partial y} \\
\frac{\partial \text{fun}(\mathbf{r})}{\partial z}
\end{array} \right]
\]
fun = magnetic field (MF) \( B \)

Linearly varying magnetic fields \( G_{x,y,z} \) in each direction

\( G \) is superposed to the static field \( (B_0) \)

\[
B(r) = B_0 \quad \Rightarrow \quad B(r) = B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z
\]

\[= B_0 + G.r\]

\[
G = \begin{bmatrix} G_x \\
G_y \\
G_z \end{bmatrix}
\]
Some examples of linearly varying MF

\[ B(\mathbf{r}) = B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z \]

\[ G_x = 3 \quad G_y = 0 \quad z = 0 \]
Some examples of linearly varying MF

\[ B(\mathbf{r}) = B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z \]

\[ G_x = 3 \quad G_y = -1 \quad z = 0 \]
Gradient of the magnetic field

\[ \nabla B(r) = \begin{bmatrix} \frac{\partial}{\partial x} \left( B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z \right) \\ \frac{\partial}{\partial y} \left( B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z \right) \\ \frac{\partial}{\partial z} \left( B_0 + G_x \cdot x + G_y \cdot y + G_z \cdot z \right) \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \\ G_z \end{bmatrix} = \mathbf{G} \]

\( \mathbf{G} \) corresponds to the gradient of the MF
The gradient coils generate \( \mathbf{G} \)
Gradient coils / Principles

Gradient coils / How it looks like?
Effect of currents

The Biot-Savart law (1820)

MF generated by a current that circulates through wires

\[ dB = \frac{\mu_0}{4\pi} \frac{dl \times r}{r^3} \]

\( l = \) current intensity

\( B \propto I \)
**Time varying fields**

Currents = independant functions of time

\[
\mathbf{I}(t) = \begin{bmatrix}
I_x(t) \\
I_y(t) \\
I_z(t)
\end{bmatrix}
\]

\[
\mathbf{G}(t) = \begin{bmatrix}
G_x(t) \\
G_y(t) \\
G_z(t)
\end{bmatrix}
\]
Effects of MF gradients on NMR signal

Time AND spatially dependent frequency

\[ B(\mathbf{r}) = B_0 + \mathbf{G.r} \quad \rightarrow \quad B(\mathbf{r},t) = B_0 + \mathbf{G(t).r} \]

Larmor equation

\[ f(\mathbf{r},t) = \frac{\gamma}{2\pi} B_0 + \frac{\gamma}{2\pi} \mathbf{G(t).r} \]

Phase of the transverse magnetization

\[ \phi(\mathbf{r},t) = 2\pi \int_{0}^{t} [f(\mathbf{r},t') - f_0] dt' = \gamma \int_{0}^{t} \mathbf{G(t').r} dt' \]
The k variable

Gradient antiderivatives

\[ \phi(r, t) = \gamma \int_{0}^{t} \mathbf{G}(t') \cdot r \, dt' = r \cdot \gamma \int_{0}^{t} \mathbf{G}(t') \, dt' = r \cdot k(t) \]

\[ k(t) = \gamma \int_{0}^{t} \mathbf{G}(t') \, dt' = \begin{bmatrix} k_x(t) \\ k_y(t) \\ k_z(t) \end{bmatrix} = \begin{bmatrix} \gamma \int_{0}^{t} G_x(t') \, dt' \\ \gamma \int_{0}^{t} G_y(t') \, dt' \\ \gamma \int_{0}^{t} G_z(t') \, dt' \end{bmatrix} \]

k is also known as the « wave vector »
Effects of MF gradients on NMR signal

Signal coming from the whole sample

\[ \tilde{S}(t) = \int \tilde{S}(r, t) dr \propto \int M_T(r) \exp[i\varphi(r, t)] dr \]

Using the « k » formulation

\[ \tilde{S}(\mathbf{r}) \propto \int M_T(r) \exp[i \mathbf{r} \cdot \mathbf{k}(t)] dr \]

\[ \tilde{S}(\mathbf{k}) \propto \int M_T(r) \exp[i \mathbf{r} \cdot \mathbf{k}] dr = \text{FT}^{-1}(M_T(r)) \]
The k-space

Acquired NMR signal in the k-space

Time-domain NMR signal is represented as function of k
Reparameterization of acquired signal

$$\hat{S}(k) \propto \text{FT}^{-1}(M_T(r))$$

$$\text{FT}[\hat{S}(k)] \propto \text{FT}[\text{FT}^{-1}(M_T(r))] = M_T(r)$$

$\hat{S}(k)$ $\text{FT}$ $M_T(r)$

k-space $\text{FT}^{-1}$ Image
$\mathcal{S}(k)$

$k$-space

$M_T(r)$

Image

$\text{FT}$

$\text{FT}^{-1}$
Remember ... $S$ is a complex number!

$z$ is orthogonal to the slide!
How riding about the k-space?

(1) Change the gradients with time

\[ \mathbf{G}(t) = \begin{bmatrix} G_x(t) \\ G_y(t) \\ G_z(t) \end{bmatrix} \]

(2) Calculate the wave-vector = integration

\[ \mathbf{k}(t) = \gamma \int_{0}^{t} \mathbf{G}(t') dt' \]
How filling the k-space?

(1) **Change the gradients** = displacement along the 3D trajectory

(2) **Acquire the signal** = fill the k-space with the (complex) value of NMR signal
How filling the k-space?

Many ways

Multi-steps / Cartesian

... and many others!
Pure « phase » encoding
Pure phase encoded imaging

Cartesian trajectory over the k-space

Gradients before the acquisition
« Phase » encoding gradient pulses
Encode the 3 directions

Whole FID available
All the chemical shift information is available!

Acquisition time
TA = Nx Ny Nz TR
    = 64.64.64. 1s = 73h
Spin Warp
Spin warp

Cartesian trajectory over the k-space

Gradients during the acquisition
« Read » gradient pulses
Encode one direction

« Phase » encoding in the 2 other directions

Acquisition time
TA = Nx Ny Nz TR
= 64.64. 1s = 1h8’
Radial encoding
RF
ACQ
$\tan \Phi = \frac{G_y}{G_x}$
Radial encoding

Radial trajectory over the k-space
Non-uniform density

Gradients during the acquisition
Only read-gradient pulses
No phase encoding

Acquisition time
\[ TA = N_\phi N_\theta TR \]
\[ = 64.64 \times 1s = 1h8' \]
Echo planar
Echo planar

(Quasi) Cartesian trajectory over the k-space

Alternated read gradients during the acquisition

Short phase encoding in-between the read gradients

Blip

Acquisition time

\[ TA = TR = 1s \]
Design of any trajectories

\[ k(t) = \gamma \int_{0}^{t} G(t')dt' \]

\[ G(t') = \frac{\partial k(t)}{\gamma \partial t} \]

Lee, Neuroimage (2010)
« Exotic » trajectories

(Some) Limits
Spatial resolution

Signal-to-noise ratio

\[ \text{SNR} \propto V = \Delta x \Delta y \Delta z \]

Voxel size

\[ \Delta x \propto k_{\text{max}}^{-1} \]

High resolution = high \( k_{\text{max}} \) = Strong gradients
Sampling

Spin warp encoding
FOV = 128 mm
16 x 16

Spin warp encoding
FOV = 128 mm
64 x 64

Object
Aliasing

Field of view (FOV) should contain the object
Solutions exist for reducing the object size
  Saturation band
  Selective pulse
  Surface coil
Off-resonance / Gradient imperfections

Off-resonance
Frequency shift $\Omega(r)$

Trajectory perturbation
Differences between experimental and theoretical gradient waveforms

$$\tilde{S}(k(t)) \propto \int M_T(r) \exp \left[ i (r \cdot k^*(t) + 2\pi \Omega(r) t) \right] dr$$

Trajectory perturbation
All the off-resonance perturbations contribute
Off-resonance

Spin warp encoding
FOV = 128 mm
16 x 16
No perturbations

With perturbations
Inhomogeneities of B1 fields

Quadrature surface coil

7 T

Transmission | B_t |

Reception | B_t |

| B^- | B^+ |

Resulting image
Homogeneous phantom

Inhomogeneities of B1 fields

Selective excitation

For reducing the number of directions to encode

Band-limited pulse
Pulse applied with a gradient pulse

Off-resonance (kHz)